



Mathematics High-Quality Instructional Materials Rubric Companion Guide

Office of Teaching and Learning

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Purpose and Organization

The companion guide breaks down each criterion from the [K-12 Mathematics High-Quality Instructional Materials \(HQIM\) Rubric](#) into smaller components for deeper analysis and offers specific examples that highlight a strong representation of that criterion might look like in practice. The examples provided in this document are not exhaustive.

This companion guide was developed in partnership with Student Achievement Partners (SAP) and is grounded in SAP's [Essential x Equitable \(e²\) Instructional Practice Framework](#). To learn more about the research and scholarship that underpins this work, please refer to [MSDE's Mathematics High-Quality Instructional Materials Identification Framework](#).

For each criterion in the Maryland Mathematics HQIM Rubric, the companion guide provides:

-  **Key Definitions:** for common language and interpretation of terms and jargon.
-  **How to Gather Evidence:** Guidance about where and how to look for evidence of each criterion.
-  **Examples of Evidence:** Offer some concrete ways criteria may appear in instructional materials. These examples are not intended to be exhaustive, but to provide reviewers with additional guidance about what evidence of criteria can look like within instructional materials.
-  **Red Flags:** Signal evidence that is not aligned or cautions of harmful implications.

Within both Examples of Evidence and Red Flags, additional guidance is provided through:

- **Grade-Level Considerations:** Where applicable, specific grade-level considerations are flagged (e.g., Grades K-2) within criteria to highlight evidence pertaining to a specific grade level or grade band. When evidence or red flags are not grade-band specific, consider them broadly applicable across grades, even if their application may look different. If grade-level considerations are nested underneath a bullet, they apply specifically to that example.
- **Student Considerations:** Where applicable, considerations for two specific student groups that are central to the design of these resources, multilingual learners and students with diverse learning needs are noted. These student considerations point to related ideas from key frameworks to support all students: CAST's [Universal Design for Learning Guidelines](#) to center students with diverse learning needs and the English Learner Success Forum's [Benchmarks of Quality](#) and [Guidelines](#) to support multilingual learners.

ORGANIZATION

Each review tool in this suite follows the same overarching organization, connected to [Maryland's Mathematics HQIM Identification Framework](#). Review materials for HQIM in Maryland are organized into four categories. Categories are divided into domains, which are further broken down into individual criteria.

For further information on the instructional materials review process, visit the [Maryland HQIM website](#).

Grade-Level and Standards Aligned

DOMAIN 1: FOCUS ON ESSENTIAL MATHEMATICS

ESSENTIAL MATHEMATICS

Instructional materials prioritize the most important and applicable concepts, knowledge, and mathematical skills. These materials include all of the following elements:

- a. a focus on the core content standards of the grade; and
- b. guidance for students and teachers to use the materials as designed and spend most of their time focused on the essential mathematics of the grade/course.

Key Definitions:

- **Core content standards:** The essential mathematical concepts and skills within each grade level in the domains of Number and Operation Sense, Algebraic Reasoning, Geometric Reasoning, and Reasoning with Data and Statistics that represent the most critical content for long-term understanding. These concepts emphasize deep comprehension over procedural techniques, ensuring students retain meaningful mathematical knowledge beyond specific formulas or methods.
- **Most of the time:** The majority of instructional time, including the focus of most units and lessons, is centered on the essential grade-level or course-level mathematical content.

How to Gather Evidence:

- Review scope and sequence to determine if essential content is prioritized.
- Review units/chapters to determine if essential content is prioritized.

Examples of Evidence:

- A scope and sequence document that outlines grade-level priorities and focuses on the essential content of the grade.
- Standards across the year are incorporated in lessons and units in a way that deepens understanding of the essential work of the grade.

Red Flags:

- Instructional materials are missing essential content or include additional content that is not aligned to grade-level or course-level standards.

DOMAIN 2: COHERENCE

CONSISTENT PROGRESSIONS

Instructional materials are consistent with the progressions in college and career-ready standards. These materials include all of the following elements:

- a. content progressions based on the grade-by-grade and course-by-course progressions in college and career-ready standards;
- b. extensive work for all students in grade-level or high school course-level problems; and
- c. grade-level or high school course-level concepts that are explicitly related to prior knowledge from earlier grades or courses.

Key Definitions:

- **Progressions:** Descriptions of topics (e.g., Ratios and Proportional Reasoning) across multiple grade levels based on children's cognitive development and the logical structure of mathematics.¹
- **Extensive work:** The vast majority (e.g., 80%) of tasks, prompts, and activities—across lessons, units, and 100% of summative assessment items—are focused on grade-level or course-level standards, ensuring that all students engage meaningfully and consistently with the mathematical content designated for their grade or course.

How to Gather Evidence:

- Review course-level maps, along with the scope and sequence, to determine whether the materials follow a logical grade-by-grade or course-by-course progression.
- Review teacher-facing materials such as unit overviews and lesson front matter for explicit guidance on how grade- or course-level concepts build on prior knowledge from earlier grades or courses.
- Examine student-facing prompts, tasks, and discussion suggestions for opportunities that explicitly engage students in activating and connecting to relevant learning from previous grades.

Examples of Evidence:

- Unit and lesson overviews that reference how current topics build on previously learned mathematics.
- Activity or task-level guidance that explains how this learning progresses, expands, and builds upon understanding over time.
- Representations and strategies that build from earlier grades or courses are carried throughout each grade as well as across program materials.
- Program or course front matter contextualizes the mathematical focus of the year within a broader vertical progression, explicitly naming how essential content develops from prior grades and prepares students for future learning.

¹ Student Achievement Partners. (2013). Progressions documents for the Common Core State Standards for Mathematics. Achieve the Core. <https://achievethecore.org/page/254/progressions-documents-for-the-common-core-state-standards-for-mathematics>

Red Flags:

- Few or inconsistent explicit references to prior knowledge, leaving students and teachers to make connections on their own without guidance.

COHERENT CONNECTIONS

Instructional materials foster coherence through connections within a single grade, or course, where appropriate and where required by college and career-ready standards. These materials include all of the following elements:

- a. supporting content to further engage students in the core content standards of the grade; and
- b. preservation of the focus, coherence, and rigor of college and career-ready standards even when targeting specific objectives.

Key Definitions:

- **Coherence:** The purposeful connection of mathematical ideas within a grade or course to support deep understanding of essential content and help students see relationships across concepts, rather than learning them in isolation.

How to Gather Evidence:

- Review unit and lesson level guidance to see how concepts and skills are developed over time.
- Review teacher materials to ensure they provide strategies for helping students see connections between concepts rather than treating them as separate topics.
- Examine lesson structure for explicit connections to previous lessons in the unit and across the year.

Examples of Evidence:

- Unit and lesson overviews that reference how current topics build.
- Teacher materials that provide guidance on making connections between concepts.
- Representations and strategies that meaningfully build throughout a unit, grade, or course.
- Explicit explanations in teacher materials on how units and sub-units build and connect.

Red Flags:

- Content from different domains presented as a separate unit rather than integrated into essential content to strengthen understanding, where applicable.
- Lessons lack explanation of mathematical connection to prior learning, making it difficult for students to see the relationships between different mathematical ideas.
- Units lack coherence, without attention to how each unit connects to prior and future learning in the grade or course.

DOMAIN 3: RIGOR AND BALANCE

RIGOR AND BALANCE

Instructional materials reflect the aspect(s) of rigor — conceptual understanding, procedural skill and fluency, and/or application — called for by the standards.² These materials include all of the following elements:

- a. development of students' conceptual understanding of key mathematical concepts, especially where called for in specific content standards or cluster headings;
- b. attention throughout the year to procedural skill and required fluencies of each grade-level³; and
- c. sufficient time for teachers and students to use the materials as designed and work with applications that engage students in problem-solving.

Key Definitions:

- **Conceptual understanding:** Building a foundation and understanding of mathematical concepts, language, operations, and relationships.
- **Procedural skill and fluency:** Developing speed and accuracy in calculation, and using procedures accurately, efficiently, and flexibly.
- **Application:** Using knowledge of math concepts and skills to solve problems in contextualized situations.

How to Gather Evidence:

- Review lessons and tasks to ensure that the three aspects of rigor are attended to with equal intensity across materials. Examine the balance of conceptual understanding, procedural skill and fluency, and application.
- Review teacher-facing materials for guidance on developing conceptual understanding and building towards fluency, as well as opportunities to make connections between strategies during application.
- Review pacing guides to ensure ample time is spent developing conceptual understanding.

Examples of Evidence:

- Tasks and activities that require students to explain their reasoning, use multiple representations, and engage in sense-making.
- Fluency practice that builds on conceptual understanding and allows opportunities for selecting and connecting appropriate strategies.

² The three aspects of rigor are not always separate in materials. (Conceptual understanding and fluency go hand in hand, fluency can be practiced in the context of applications, and brief applications can build conceptual understanding.) Nor are the three aspects of rigor always together in materials. (Fluency requires dedicated practice. Rich applications cannot always be shoehorned into the mathematics topic of the day. And conceptual understanding will not always come along unless explicitly taught.)

³ Student Achievement Partners. (n.d.). Mathematics focus by grade level. Achieve the Core.
<https://achievethecore.org/category/774/mathematics-focus-by-grade-level>

- Application tasks that present students with meaningful, real-world contexts in which interpretation of the situation is necessary to determine a solution path, rather than the path being explicitly given.
- Application tasks that are sufficiently complex to support the modeling process (e.g., making assumptions, selecting tools, refining strategies) as required by the grade or course standard.
- Application tasks require students to communicate both their solution and reasoning.
- Instructional materials include dedicated fluency routines (e.g., daily practice activities, games, or fluency warm-ups) that are intentionally sequenced and designed to build grade-level fluencies over time.
- Tasks and problem sets throughout the curriculum integrate opportunities for students to apply procedural skills within multi-step problems.

Red Flags:

- Tricks and shortcuts included in lessons and teacher notes, such as the butterfly method, that strip away conceptual understanding.⁴
- Lessons and tasks that focus exclusively on procedural skill and fluency without opportunities to connect to the underlying concepts and contexts.
- Activities that address conceptual understanding are only cursory or a means to then introduce an algorithm or procedure.
- Minimal tasks that allow students to engage and use mathematics in contextualized problems.
- Application tasks that are unrealistic, such as unreasonable quantities for the context or measuring in units that are uncommon or unrealistic. (e.g., juice measured in pounds).

⁴ Cardone, T. (2015). Nix the tricks: A guide to avoiding shortcuts that cut our math concept development. (2nd ed.). <https://www.nixthetricks.com/NixTheTricks2.pdf>

DOMAIN 4: MATHEMATICAL PRACTICES

PRACTICE-CONTENT CONNECTIONS

Instructional materials meaningfully integrate Standards for Mathematical Practice with content standards and attend to the full meaning of each practice standard in tasks and problems.

Key Definitions:

- **Standards for Mathematical Practice (SMPs):** The set of eight practices that describe the habits of mind and actions students should develop to engage deeply with mathematics. These practices emphasize problem-solving, reasoning, communication, and the application of mathematical concepts to build understanding and proficiency.⁵

How to Gather Evidence:

- Review lessons and tasks to evaluate how the Standards for Mathematical Practice (SMPs) are integrated into the mathematical content.
- Examine teacher-facing materials for guidance on addressing the full meaning of each practice standard and understanding how it connects to the grade-level mathematics.

Examples of Evidence:

- Teacher materials that include prompts, scaffolds, and strategies for emphasizing SMPs within content learning, such as questions to encourage the use of structure (e.g., SMP 7).
- Tasks and activities that provide opportunities for students to reflect on their use of the SMPs (e.g., constructing arguments and critiquing reasoning) in relation to the mathematical content.

Red Flags:

- Lack of teacher guidance for connecting SMPs with content learning or superficial references to SMPs such as references to the Mathematical Teaching Practices.⁶
- An overabundance of references or tags to the Standards for Mathematical Practice within a single lesson or unit that dilutes the focus and hinders intentional preparation for engaging meaningfully with a manageable number of practices.
- Limited or inconsistent references to the Standards for Mathematical Practice, resulting in missed opportunities for students to develop key mathematical habits of mind through sustained engagement.
- Materials repeatedly highlight some practices (e.g., "constructing arguments" or "modeling") while neglecting others (e.g., "attending to precision" or "using structure"), resulting in an imbalanced approach to mathematical practice development.

⁵ Maryland State Department of Education. (2022). Standards for mathematical practice. <https://marylandpublicschools.org/about/Documents/DCAA/Math/StandardsForMathematicalPractices.pdf>

⁶ National Council of Teachers of Mathematics. (2014). Principles to actions: Ensuring mathematical success for all. <https://www.nctm.org/Conferences-and-Professional-Development/Principles-to-Actions-Toolkit/Resources/7-EffectiveMathematicsTeachingPractices/>



EMPHASIS ON MATHEMATICAL REASONING AND MATHEMATICAL MODELING

Instructional materials support the standards' emphasis on mathematical reasoning and mathematical modeling through indicating and providing guidance about the opportunities for discourse, communication, and problem-solving.

Key Definitions:

- **Mathematical reasoning:** Applying logical and critical thinking to understand and solve problems, make connections between concepts, and justify mathematical ideas—moving beyond rote memorization toward deeper understanding.
- **Mathematical modeling:** A process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena.

How to Gather Evidence:

- Review lessons for activity structures (e.g., instructional routines) that explicitly prompt mathematical reasoning, such as problem-solving, modeling, and constructing arguments.
- Examine teacher materials for guidance on facilitating student discourse and encouraging reasoning during instruction.
- Review tasks for opportunities where students explain, justify, and critique solutions.

Examples of Evidence:

- Tasks and activities that require students to explain their reasoning, justify solutions, or critique the reasoning of others.
- Teacher prompts, questions, or guides specific to the lesson content that allows for teachers to enrich the lesson and facilitate deeper reasoning.
- Instructional routines, discussion protocols, or activity structures that encourage discussion and prioritize reasoning, such as turn and talk or math talks.
- Prompts and sentence frames for sharing reasoning with peers.
- Tasks that embody the mathematical modeling process⁷, including:
 - Beginning with real-world scenarios requiring students to formulate a mathematical question based on the context (e.g., deciding how to price a product or analyze a public health trend);
 - Opportunities for students to make assumptions, define variables, and select appropriate representations (e.g., graphs, equations, simulations) to model a real-world situation; and
 - Guidance for teachers and students to revise or refine models based on limitations, new information, or the reasonableness of results.

⁷ Solow, D., & Turner, H. (Eds.). (2016). GAIMME: Guidelines for assessment and instruction in mathematical modeling education (2nd ed.). Consortium for Mathematics and Its Applications (COMAP) & Society for Industrial and Applied Mathematics (SIAM). https://www.comap.com/images/resources/free_resources/GAIMME_Report.pdf

Red Flags:

- Limited guidance for teachers on how to facilitate mathematical reasoning.
- Opportunities for reasoning are primarily independent or through teacher-student interactions, lacking student to student opportunities for discourse on reasoning.

Designed to Affirm Students

DOMAIN 1: CULTURALLY RESPONSIVE-SUSTAINING INSTRUCTION

AFFIRMATION AND CENTERING OF STUDENTS

Instructional materials are designed to encourage students to anchor learning in individual experiences, backgrounds, and cultural knowledge to expand their mathematics knowledge and skills.

Key Definitions:

- **Culturally responsive-sustaining instruction:** An approach to teaching that recognizes, affirms, and builds on students' cultural, linguistic, and lived experiences as strengths, and creates learning environments that sustain students' identities while promoting academic success, critical thinking, and social awareness.⁸

How to Gather Evidence:

- Identify tasks with opportunities where students deepen, reflect on, or explore their cultural knowledge and individual experiences in relation to mathematics.
- Review call-out boxes, prompts, or embedded professional learning in teacher materials that build understanding of culturally responsive-sustaining instruction, including how to affirm and incorporate students' cultural, linguistic, and community knowledge into math teaching.
- Examine the names, traditions, and situations in contextualized problems for the extent to which they reflect a variety of identities.

Examples of Evidence:

- Prompts and tasks that allow students to share how their personal experiences or cultural backgrounds inform their understanding of math.
- Teacher guidance on integrating cultural knowledge into math instruction.
- Contextualized problems affirm positive math identities for all students by including a balanced representation (e.g., race, ethnicity, gender, ability, etc.).
- Tasks and teacher supports that highlight mathematical discoveries and contributions from a diverse range of mathematicians, including but not limited to Black, Indigenous, Brown, and non-Western individuals, to reflect a broad and representative history of mathematics.⁹

⁸ New York State Education Department. (2019). Culturally responsive-sustaining education framework. <https://www.nysed.gov/sites/default/files/programs/crs/culturally-responsive-sustaining-education-framework.pdf>

⁹ Harris, B., Herring, W., & Stone, R. (2024, February 12). Supporting culturally responsive teaching practices through math curricula. Mathematica, Inc. <https://www.mathematica.org/download-media?MediaItemId=%7BD1A37EF8-A083-4444-A48C-9C03FC57E57D%7D>



Red Flags:

- Materials that treat cultural knowledge superficially or in a tokenistic way rather than central to math learning.

MATHEMATICS AS A TOOL FOR CIVIC ENGAGEMENT

Instructional materials consistently include tasks that prompt students to apply and develop their civic engagement skills and examine social contexts and current events, using mathematics to question the world and the current status quo.

Key Definitions:

- **Civic engagement:** Active participation in one's community and society through critical thinking, dialogue, and action—using mathematics to analyze social issues and contribute to informed decision-making and positive change.

How to Gather Evidence:

- Examine teacher materials for guidance on helping students use math as a tool to make sense of current events or take civic action.
- Review tasks for opportunities where students engage in collaborative discussions about social and civic issues through mathematical reasoning.
- In lessons, review and identify opportunities for students to critically analyze dominant narratives, question social inequities, and engage in contextualized problem-solving tied to their communities.

Examples of Evidence:

- Tasks that analyze social justice issues (e.g., income inequality, voting patterns) using real-world data and mathematical models.
 - **Grades K-2:** Tasks that explore fairness, sharing resources, or making group decisions using simple data collection and visual models (e.g., tally charts, bar graphs).
- Materials include examples of students or peers using mathematics to analyze and address real-world issues in their communities.
- Collaborative projects and tasks that allow flexibility for students to explore and create solutions to local or global issues—such as community resource distribution or climate change, making mathematical learning relevant to their own contexts.
 - **Grades K-2:** Projects may focus on familiar, tangible issues (e.g., organizing a class event, creating a weather chart to decide what to wear), allowing students to use counting, comparison, and measurement to make decisions as a group.
- Teacher resources that provide strategies for facilitating discussions, specifically supporting students to use mathematics to think critically about their world, experiences, and current events.
- Discussion prompts or activities that encourage students to analyze community issues, reflect on dominant and absent voices, and propose solutions or actions.
- Tasks and prompts that support students in critically analyzing data sources, including recognizing when data should be verified with additional or more reliable sources.
- Opportunities for students to examine how context influences the interpretation of data, including how data can be used to support or challenge claims related to social and community issues.

Red Flags:

- Tasks or prompts present a single, predetermined viewpoint or conclusion about a social issue, limiting students' opportunities to analyze data, ask questions, and develop their own mathematical claims or interpretations.

REAL-WORLD CONNECTIONS AND RELEVANT DATA

Instructional materials consistently connect with students' lives, their future goals, their communities, and the world and nurture ways to engage in their own communities and beyond. These materials include all of the following elements:

- a. use of mathematical concepts and tasks to connect to current events;
- b. collaborative tasks and/or projects that involve real-world problem-solving through meaningful interactions with peers and their local communities;
- c. structures (e.g., tasks, classroom activities, routines, assignments) to explore mathematical concepts from current events and data that are relevant to students' lives and communities so that students see themselves in the tasks and understand how the tasks relate to their context and promote a sense of belonging;¹⁰ and
- d. teacher guidance to support students in developing mathematical skills and knowledge that are relevant to their academic and professional goals.

Key Definitions:

- **Current events:** Timely issues, situations, or topics that are relevant to students' communities or the broader world. Instructional materials are not expected to reference real-time or up-to-date news but should include prompts, structures, or examples that guide teachers and students to make meaningful connections between mathematical content and current or emerging issues in their local or global contexts.

How to Gather Evidence:

- Review tasks, projects, and classroom activities for explicit connections to students' lives, communities, and/or real-world contexts, including current events.
- Review teacher materials for guidance on incorporating meaningful interactions between students and their communities through real-world problem-solving.

Examples of Evidence:

- Activities or projects that use real-world data (e.g., community statistics, climate data, etc.) to solve problems related to current events.
- Activities, routines, or projects that give students opportunities to see themselves in the tasks and understand how the tasks relate to their context and promote a sense of belonging.
- Prompts or teacher suggestions to connect the mathematical content of the materials to current events (e.g., How might understanding this math concept help you make decisions or solve problems?).
- Tasks and problem contexts that are developmentally appropriate and designed to engage students' interests (e.g., using scenarios like sports statistics, social media trends, or budgeting for events relevant to students' age and daily experiences), reinforcing the relevance of math to their lives.

¹⁰ Tate, W. F. (1995). Returning to the root: A culturally relevant approach to mathematics pedagogy. *Theory Into Practice*, 34(3), 166–173.

- Opportunities for students to collect relevant data in their classroom, school, or community.
- Prompts to support students in making connections to data and context.
- Tasks and contexts are developmentally appropriate and reflect topics that are interesting and engaging to students (e.g., school life, sports, environmental issues).
- Prompts support connections to real-world problems and provide opportunities for students to pose questions about their experiences or communities that can be explored using mathematics.

Red Flags:

- Opportunities to examine real world data are superficial or disconnected from student's lives.

DOMAIN 2: LANGUAGE AFFIRMING INSTRUCTION

MULTILINGUALISM IN MATHEMATICS

Instructional materials are deliberately designed to honor and build upon students' language(s) as an asset, encouraging students to use their linguistic repertoire to communicate with one another via reading, writing, speaking, and listening while engaging in mathematical learning. These materials include all of the following elements:

- a. providing facilitation and engagement support for students to communicate as they do the math, make meaning, and collaboratively solve problems;
- b. building mathematical language and content in English and home language(s),^{11 12} including use of social and academic vocabulary, through translanguaging so all students express themselves in a language they are comfortable with while working to learn mathematical content and meet language objectives in the target language;¹³ and
- c. making cross-linguistic connections, including identifying and comparing similarities and differences between home language(s) and English (e.g., cognates) or registers and registers of instruction.

Key Definitions:

- **Home language(s):** The language(s) students use in their families and communities to communicate, make meaning, and learn. Instructional materials are not expected to be translated into all world languages but should include teacher-facing supports that elevate and build connections to students' home language(s), such as identifying cognates, supporting translanguaging, and acknowledging linguistic similarities and differences to deepen mathematical understanding.
- **Translanguaging:** Practice of using multiple languages flexibly and strategically for fluid communication, which includes use of students' home language(s) and their full linguistic repertoire.

How to Gather Evidence:

- Review tasks and materials for embedded supports that allow students to use their linguistic repertoire (e.g., translanguaging, social and academic vocabulary building).
- Examine teacher-facing materials for guidance on making cross-linguistic connections and building on students' home languages.

Examples of Evidence:

- Materials highlight similarities and differences between students' home languages and English, such as identifying cognates or exploring varying mathematical terminology across languages, to deepen understanding and support language development.

¹¹ English Learners Success Forum. (n.d.). Math guideline area of focus 1: Interdependence of Mathematical Content, Practices, and Language. <https://www.elsuccessforum.org/math-guidelines/math-area-of-focus-1>

¹² English Learners Success Forum. (n.d.). Math guideline area of focus 4: Leveraging students' assets. <https://www.elsuccessforum.org/math-guidelines/math-area-of-focus-4>

¹³ García, O., Johnson, S. I., & Seltzer, K. (2017). The translanguaging classroom: Leveraging student bilingualism for learning. Caslon.



- Guidance for teachers to facilitate sustained, back-and-forth peer discussions and collaborative problem-solving in ways that validate and build on students' languages (e.g., using mathematical language routines¹⁴ such as Collect and Display).
- Opportunities for students to engage in communication with clear, purposeful goals (e.g., explaining reasoning, critiquing a peer's solution, or co-constructing strategies) through oral, written, and visual methods in their preferred language(s).
- Opportunities for students to use oral, written, and visual methods to communicate mathematical ideas in their preferred language(s).
- Lessons and activities that encourage and highlight connections between home languages and English, including incorporating translanguaging strategies in order to enable students to discuss mathematical concepts in the language they are most comfortable with while simultaneously developing proficiency in the target language.
- Structures and supports (such as sentence frames and mathematical language routines) that foster equitable participation and encourage risk-taking in mathematical conversations across languages.

Red Flags:

- Materials that treat students' home languages as barriers rather than assets for learning mathematics (e.g., over-emphasis on one right way of communicating).
- Limited or overly generic guidance for teachers.

¹⁴ Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). Principles for the Design of Mathematics Curricula: Promoting Language and Content Development. Retrieved from Stanford University, UL/SCALE website. https://ul.stanford.edu/sites/default/files/resource/2021-11/Principles%20for%20the%20Design%20of%20Mathematics%20Curricula_1.pdf

LANGUAGE OBJECTIVES

Instructional materials provide explicit alignment between language and content objectives to ensure that the language goals embedded within the standards are being attended to in every lesson. This includes language objectives for both expressive (writing and speaking) and receptive (listening and reading) communication that are aligned to the math learning goal.

Key Definitions:

- None

How to Gather Evidence:

- Review lessons to ensure alignment between language and content objectives for both expressive (speaking, writing) and receptive (listening, reading) communication.
- Review teacher materials for explicit language objectives that are integrated with mathematical goals.

Examples of Evidence:

- Lessons with clearly stated language and content objectives, including identification of expressive and/or receptive communication objectives (e.g., "Students will explain their reasoning using sentence stems that include academic vocabulary such as 'justify' and 'evaluate.' (Speaking)").
- Tasks that incorporate strategies for expressive and receptive communication, such as partner discussions or written reflections.
- Teacher-facing materials with guidance on scaffolding both mathematical content and language development (e.g., coupling exploratory lessons/activities with opportunities to use and celebrate home and informal language).
- Language-based supports, such as writing prompts or mathematical language routines,¹⁵ that are directly applicable to core instruction.

Red Flags:

- Lessons with no explicit language objectives or misalignment between language and math goals.
- Limited opportunities to incorporate practice with both receptive and expressive communication, or over-emphasis of one over the other.
- Overly general language objectives that are repeated throughout lessons without being tailored to specific math content.

¹⁵ Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). Principles for the Design of Mathematics Curricula: Promoting Language and Content Development. Retrieved from Stanford University, UL/SCALE website. https://ul.stanford.edu/sites/default/files/resource/2021-11/Principles%20for%20the%20Design%20of%20Mathematics%20Curricula_1.pdf

COGNITIVELY DEMANDING MATHEMATICS

Instructional materials provide cognitively demanding mathematics tasks that offer multiple research-based entry points and linguistic scaffolds to meet the needs of multilingual learners and students with diverse learning needs.

Key Definitions:

- **Linguistic scaffolds:** Supports embedded in instruction that help students access and express mathematical thinking—such as sentence frames, word banks, visuals, labeled diagrams, and structured discussion routines—while developing academic language and content understanding.

How to Gather Evidence:

- Review tasks to ensure they offer multiple entry points, such as tasks with a low floor/high ceiling, and include linguistic scaffolds to support all learners in accessing and expressing their mathematical thinking.
- Identify opportunities in lessons where students share reasoning through various modes, such as oral explanations, written work, and visual representations.

Examples of Evidence:

- Teacher-facing guidance that highlights opportunities for students to use language functions (explaining, justifying, critiquing) while engaging in grade-level mathematical tasks, and to revise their own or peers' mathematical explanations for clarity and precision.
- Tasks that require students to engage in higher-order thinking, such as problem-solving, analyzing patterns, and mathematical modeling.
- Teacher-facing guidance for scaffolding tasks cognitively demanding tasks, including when and how to support productive struggle without reducing grade-level expectations.
- Examples of student work in teacher-facing materials that demonstrate reasoning through multiple representations, such as diagrams, equations, and verbal explanations.
- Guidance on implementing anchor charts, visual aids, and models as resources for students to reference during problem-solving and reasoning.
- Opportunities in teacher materials for students to analyze and address mathematical errors or misconceptions, with guidance for teachers on facilitating these conversations.
- Tasks include linguistic scaffolds—such as sentence frames, word banks, visuals, and structured discussion supports—that help students access and communicate their mathematical thinking while engaging in cognitively demanding tasks.

Red Flags:

- Scaffolds or supports reduce the cognitive demand of tasks to below grade-level expectations, limiting opportunities for all students to engage meaningfully in cognitively demanding, grade-appropriate mathematics.
- Activities that are classified as “exploratory” or “problem-solving” that are overly procedural, lack opportunities for deep thinking and exploration, or value one right way.

REASONING IN MULTIPLE WAYS

Instructional materials include tasks that invite students to share their reasoning in multiple ways and guidance (e.g., annotations for teachers facilitating the tasks) about encouraging students to transverse between and among different representations (e.g., oral language and pictorial representations, written word and math tools).

Key Definitions:

- None

How to Gather Evidence:

- Review tasks for opportunities where students are encouraged to share reasoning in multiple ways (e.g., orally, pictorially, symbolically).
- Review teacher-facing materials for notes or strategies to support students in connecting and moving between representations.

Examples of Evidence:

- Tasks with explicit prompts for students to use different representations, such as creating a graph, explaining orally, and writing equations.
- Teacher materials with guidance on encouraging students to make connections between representations and justify their reasoning, such as prompts to help students make connections between concrete and abstract representations.
- Instructional routines or tasks where students present their thinking in multiple formats, critique peers' methods, or explore alternative reasoning.

Red Flags:

- Lessons and units focus exclusively on a single mode of mathematical representation (e.g., only using equations or only using diagrams).
- Tasks emphasize a single strategy or solution path, rather than encouraging students to explore, compare, or justify multiple valid approaches to solving a problem.

Instructional Design

DOMAIN 1: STUDENT AGENCY

METACOGNITIVE PROCESSES

Instructional materials develop students' metacognitive skills to promote understanding of math concepts by directly teaching and supporting students to monitor understanding and progress over time. These materials include all of the following elements:

- a. setting goals, self-monitoring growth, and reflecting on the impact of students' choices and ongoing development as mathematical doers, critical thinkers, and communicators;
- b. providing explicit practices to develop students' metalinguistic awareness around how language works in mathematics, language use, and choices connected to mathematical ideas;
- c. modeling and developing strategies that support students in making their thinking visible through speaking, writing, or drawing their developing understanding; and
- d. supporting students with diverse learning needs in developing metacognitive strategies.

Key Definitions:

- **Metacognitive skills:** Set of skills that support students in actively monitoring, planning for, and adapting their learning and reasoning process.
- **Metalinguistic awareness:** Consciously attending to, reflecting on, and addressing the features of language.

How to Gather Evidence:

- Review front matter, unit overviews, and lessons for metacognitive process supports through teacher modeling, questioning, student routines (including goal-setting cycles), or student tools/resources. Identify opportunities for students to set goals and monitor progress toward meeting them.
- Review language supports in unit overviews, lessons, and call out boxes for the presence of explicit time and meaning-making activities to build metalinguistic awareness.
- Review unit overviews, lessons, and supporting resources (e.g., graphic organizers, reflection prompts) for opportunities to guide students to organize their thoughts and monitor their understanding.

Examples of Evidence:

- Clear guidance within lessons to model and support students' use of metacognitive strategies in mathematical thinking, such as self-questioning, summarizing key concepts, clarifying problem-solving approaches, regulating attention, and organizing mathematical ideas within the context of mathematical tasks and discussions.
- Teacher-facing guidance includes clear instruction and support for helping students make sense of the symbolic language of mathematics, such as equations and notation, and highlights that math is a distinct language that requires decoding and interpretation.

- Make thinking visible to students through think aloud strategies, use of metalinguistic questioning (e.g., Teacher-facing materials that make thinking visible through think-aloud strategies and metalinguistic questioning (e.g., “How is ‘difference’ related to subtraction?”), combined with prompts and feedback structures that encourage students to explain and adjust their thinking.¹⁶
- Student materials include specific, lesson-embedded supports for goal setting, self-monitoring, and self-reflection in mathematics. These resources provide students with tools such as models, rubrics, checklists, and self-reflection templates to assess and adjust their mathematical reasoning, problem-solving strategies, and participation in mathematical discussions.¹⁷

Red Flags:

- Self-monitoring strategies are suggested or modeled by the teacher without connections to authentic student reflection, application, and ownership (e.g., focus on teacher-driven feedback and not student reflection; “Watch me as I...”).
- Assessments are not connected to opportunities to build metacognitive awareness and skills (e.g., focus on summative assessments without formative assessments; “Here is your grade”).

¹⁶ CAST (2024). Universal Design for Learning Guidelines version 3.0, Action & Expression, Strategy Development, Consideration 6.2: Anticipate and plan for challenges. <https://udlguidelines.cast.org/action-expression/strategy-development/challenges/>

¹⁷ CAST (2024). Universal Design for Learning Guidelines version 3.0, Action & Expression, Strategy Development, Consideration 6.4: Enhance capacity for monitoring progress. <https://udlguidelines.cast.org/action-expression/strategy-development/monitoring-progress/>

CHOICE

Instructional materials prompt teachers to provide students ample time to explore math concepts, during which students are given opportunities to make choices about how to spend time, whom to spend it with, and what materials are used.

Key Definitions:

- None

How to Gather Evidence:

- Review lessons for activities and tasks that create opportunities for student choice and feedback within daily instruction.
- Review assessments across units and grade levels, specifically at the introduction and culmination of a topic, for opportunities for students to choose ways to demonstrate learning, explore a new idea, or review and practice in a variety of ways.

Examples of Evidence:

- Teacher-facing guidance on identifying opportunities to strategically provide choice to students.
- Clear and lesson-embedded opportunities for students to make self-selected choices in daily instruction through topic or task. This includes opportunities for student choice in selecting which materials, strategies, manipulatives, and numbers to use.
- Prompts for students to reflect on the utility, efficiency, and effectiveness of their instructional choices (e.g., “How did the tool you chose help you understand the math?” “Was your strategy efficient?”).
- Tasks and prompts that provide opportunities for students to share, explore, and use mathematics to learn about their curiosities or pursue their own questions.
- Options for choosing methods for expressing students’ understanding that best reflect their strengths as learners and their understanding of the content, for example, a project might give students the opportunity to share their findings in a recorded podcast.
- Unit plans and lessons include multiple representations and ways students might represent the tasks with a variety of manipulatives, models, or solution paths.

Red Flags:

- Opportunities for student choice interfere with students’ ability to engage with grade-level content and tasks, and the opportunities are not comparatively rigorous or worthy of students’ time.
- Choice opportunities are only ever framed as “extension” work and not a part of core instruction.

MULTIPLE ENTRY POINTS FOR COMPLEX TASKS

Instructional materials include tasks that are complex, with multiple entry points (e.g., allow for multiple solution strategies, encourage use of multiple representations) that promote collaboration and different ways of thinking and explaining.

Key Definitions:

- None

How to Gather Evidence:

- Review lesson activities and tasks that allow for multiple entry points, support multiple solution strategies, and encourage diverse representations.
- Review lesson activities and tasks for a variety of collaborative norms and structures. (e.g., think-pair share, partner work, small group work, independent think time).
- Review teacher-facing materials for guidance on fostering collaboration and supporting students to engage authentically as mathematicians.

Examples of Evidence:

- Tasks that explicitly support multiple entry points and solution strategies, with opportunities for students to use representations such as diagrams, equations, or verbal explanations.
- Samples of student work, possible solutions, or reflections showing varied ways of thinking and explaining the mathematics.
- Guidance in teacher materials, as well as prompts in student-facing materials, to facilitate collaboration and discussions about different approaches to solving problems.

Red Flags:

- Tasks are overly procedural, with a single prescribed solution path or representation.
- Lack of teacher guidance for promoting collaboration or supporting diverse approaches.
- Collaboration opportunities are superficial or do not enhance students' engagement with mathematical concepts.

AUTHENTIC ENGAGEMENT AS A MATHEMATICIAN

Instructional materials promote productive struggle and the mathematical modeling process through quality math tasks that are sequenced to build conceptual understanding and procedural skill and fluency, prioritize inquiry, provide opportunities to take risks, allow for rough draft thinking and multiple approaches, invite the use of math tools, and use mistakes for learning so that students engage in collaborative learning.

Key Definitions:

- **Productive struggle:** A process where students actively engage with challenging tasks, even when encountering obstacles.
- **Mathematical modeling process:** A systematic process of formulating questions, translating a contextualized problem into a mathematical representation, identifying relevant variables, making assumptions, developing and applying mathematical methods to find solutions, interpreting results, and validating the solution within the original context to gain insights and make predictions about the system being studied.
- **Inquiry:** Opportunities for students to explore, question, and discover mathematical ideas through problem-solving, reasoning, and sense-making rather than simply following prescribed procedures.
- **Rough draft thinking:** The practice of sharing initial, incomplete, or evolving mathematical ideas — through words, drawings, or models — with the understanding that thinking will be refined and improved through discussion, feedback, and continued reasoning.¹⁸

How to Gather Evidence:

- Identify lesson activities and tasks for opportunities that encourage risk-taking, productive struggle, and mathematical modeling while supporting rough draft thinking (e.g., dedicated time to explore topics).
- Review teacher-facing materials (e.g., front matter, unit overview, subunit overview, professional learning modules) for guidance on using mistakes as learning opportunities and strategies for fostering inquiry-based discussions that encourage students to construct viable arguments and critique the reasoning of others.
- Review the table of contents, sequence of lessons and tasks within units to assess how concepts build understanding and fluency over time.
- Review introductory lessons for the year or course to see how a risk-taking community is built.

Examples of Evidence:

- Tasks that require students to engage in mathematical modeling, use math tools strategically, and experiment with multiple approaches.

¹⁸ Jansen, A. (2020, March 9). What is rough draft math? University of Delaware UDaily.
<https://www.udel.edu/udaily/2020/march/rough-draft-math-amanda-jansen-education-human-development/>

- Activities that support rough-draft thinking by prompting students to explore ideas, refine their reasoning, and iterate on solutions through discussion, reflection, and revision (e.g., the Critique, Correct, Clarify routine¹⁹).
- Lessons and tasks intentionally increase in complexity to build student stamina.
- Explicit lessons that support establishing a math community.
- Professional reading on creating a classroom culture that rewards risk-taking.
- Task-level teacher guidance for supporting students in identifying an entry point.

Red Flags:

- Limited opportunities for inquiry, risk-taking, or collaborative learning.
- Opportunities for mathematical modeling are superficial or only available through extension activities.

¹⁹ Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). Principles for the Design of Mathematics Curricula: Promoting Language and Content Development. Retrieved from Stanford University, UL/SCALE website. https://ul.stanford.edu/sites/default/files/resource/2021-11/Principles%20for%20the%20Design%20of%20Mathematics%20Curricula_1.pdf

COLLABORATIVE LEARNING

Instructional materials engage all students in collaborative learning through a variety of research-based routines, structures, and tasks that allow for whole-group, small-group, and independent thinking. Materials explicitly plan for students to demonstrate their curiosity and share their tentative thinking; ask questions; and adjust their understanding by listening to and building on one another's shared ideas.

Key Definitions:

- **Collaborative learning:** Learning, problem-solving, and meaning-making through interactions among peers.

How to Gather Evidence:

- Review lessons, games, centers, and activities for opportunities for students to learn collaboratively through sharing, listening, revoicing, writing.
- Review lessons, games, centers, and activities for embedded routines and structures that support collaborative learning and are taught to students and utilized throughout the units and program.
- Review teacher guidance for explicit support and lesson-embedded structures to facilitate collaborative learning and encourage the sharing of tentative thinking.

Examples of Evidence:

- Frequent opportunities for student collaboration that include a variety of structures (e.g., frequent partner conversations, extended group-learning activities, think-pair-share before whole-group discussion, different roles in collaborative groups).
- Opportunities for collaborative problem solving and modeling as well as co-constructing reasoning.
- Tasks that allow students to work with peers to reach a common goal.
- Opportunities for students to revisit their thinking over the course of a lesson and a unit after interacting with peers (e.g., returning to anchor charts, key questions, or student work).
- Guidance, tools, and/or prompts for students in supporting one another through challenging academic tasks and asking for help as needed (e.g., sample teacher prompts and modeling, sample student prompts, intentional opportunities for pausing to share challenges and potential strategies).²⁰

Red Flags:

- Limited guidance for supporting students during collaborative learning time (e.g., "Ask students to discuss in small groups" vs. including guidance on roles, timing, guiding questions or in-the-moment supports).

²⁰ CAST (2024). Universal Design for Learning Guidelines version 3.0, Engagement, Sustaining Effort and Persistence, Consideration 8.3: Foster collaboration, interdependence, and collective learning. <https://udlguidelines.cast.org/engagement/effort-persistence/collaboration/>

DOMAIN 2: MONITORING PROGRESS AND SUPPORTING STUDENTS

SUPPORTS AND SCAFFOLDS²¹

Instructional materials apply a research-based approach to develop students' metacognition by directly teaching and supporting students to monitor understanding during reading and self-regulate during writing. These materials include all of the following elements:

- a. scaffolds and supports that are designed based on mathematical learning progressions and the coherence of math concepts across and within grades and courses;
- b. guidance on identifying scaffolds and appropriate supports that build on students' mathematical thinking, ideas, and experiences; and
- c. content-specific guidance on identifying and addressing potential individual student needs so that supports, scaffolds, and extensions can be effectively differentiated, including adjustments to content, process, or product.

Key Definitions:

- **Content:** Differentiating tasks, activities, or resources (in ways that do not interfere with students' consistent opportunity to engage with grade-level content).
- **Process:** Differentiating how students engage with the content (e.g., collaboration or learning preferences).
- **Product:** Differentiating the ways students demonstrate their learning of grade-level content.

How to Gather Evidence:

- Review teacher guidance, unit or chapter overviews, or supplementary materials for information about how to implement provided supports, extensions, and scaffolds (e.g., intervention supports, differentiation supports, supplemental guides) for diverse student learning needs.
- Review identified resources that provide reteaching of skills and concepts for students who have not yet reached grade-level content and skills, (e.g., call out boxes, modifications to tasks, teacher tips).
- Examine core and supporting resources for enrichment and extension activities designed to challenge students who are exceeding grade-level content and skills.
- Compare how supports and scaffolds are treated in an early-year unit and a later unit for shifts in responsibility to students and how scaffolding is approached from year-to-year.

²¹ Thoughtfully designed questions and tasks that provide access to grade-level, culturally responsive-sustaining, and language-affirming experiences for students are one form of support for students and are addressed in other sections of this framework.

Examples of Evidence:

- Standards-aligned scaffolds and supplemental resources that offer additional support or challenge for students based on their learning needs.²²
- Scaffolds and supplemental activities that are aligned to appropriate aspect of rigor and learning progressions, for example, leveraging representations and models from previous courses and/or units to reinforce new learning.
 - **Grades K-2:** Using place value charts as students move from adding/subtracting within 20 to adding/subtracting within 100.
 - **Grades 6-7:** Connecting tape diagrams and equations as students move from representing situations with equations in the form of $px = r$ to $px + q = r$, for example.
- Task-specific guidance on:
 - Utilizing supports and scaffolds within each lesson, including a focus on connecting to previous key ideas and guiding the processing of information.^{23 24}
 - Methods of adapting provided content to meet student needs (e.g., alternative teaching approaches, pacing, instructional delivery option).
 - Gradual release of supports and scaffolds over the course of a series of lessons or unit, beginning with models and releasing to more independence (e.g., first lesson providing discussion of prior content learning; providing exemplars or peer mentors and transitioning to checklists or templates).²⁵
- Supplemental resources specifically designed to support re-engagement opportunities (e.g., manipulatives, technology enhancements, small-group materials, visual aids).
- Supplemental resources, tasks, and recommendations designed to provide additional challenge by deepening students' understanding of topics under study (e.g., extension or additional tasks aligned to topic).
- Content-specific resources to support learning differences for just in time supports and development.
- Supplemental materials and lessons include the same language, models, and strategies used in the core materials.

²² CAST (2024). Universal Design for Learning Guidelines version 3.0, Action & Expression, Expression & Communication, Consideration 5.3: Build fluencies with graduated support for practice and performance. <https://udlguidelines.cast.org/action-expression/expression-communication/fluencies-practice-performance/>

²³ CAST (2024). Universal Design for Learning Guidelines version 3.0, Representation, Building Knowledge, Consideration 3.1: Connect prior knowledge to new learning. <https://udlguidelines.cast.org/representation/building-knowledge/prior-knowledge/>

²⁴ CAST (2024). Universal Design for Learning Guidelines version 3.0, Representation, Building Knowledge, Consideration 3.4: Maximize transfer and generalization. <https://udlguidelines.cast.org/representation/building-knowledge/transfer-generalization/>

²⁵ CAST (2024). Universal Design for Learning Guidelines version 3.0, Action & Expression, Expression & Communication, Consideration 5.3: Build fluencies with graduated support for practice and performance. <https://udlguidelines.cast.org/action-expression/expression-communication/fluencies-practice-performance/>



Red Flags:

- Scaffolds and supports that are formulaic and do not attend to the specific content, skills, or tasks at hand.
- Scaffolds and supports reduce the rigor of the task (e.g., simplifying the content demands instead of amplifying them).
- Materials and guidance only provided for reteaching (doing the same things again, which is often less effective) as opposed to reengaging (teaching in a new way, which is preferred)
- Supplemental lessons do not attend to learning progressions.

SIMULTANEOUS MATHEMATICAL MEANING-MAKING AND LANGUAGE DEVELOPMENT

Instructional materials include intentional language learning opportunities alongside appropriate, research-based supports for multilingual learners and students with diverse learning needs to develop mathematical meaning-making and language simultaneously. Materials include questions for students to raise metalinguistic awareness of how language works in math and integrate language standards alongside mathematical content standards.²⁶

Key Definitions:

- **Metalinguistic awareness:** Consciously attending to, reflecting on, and addressing the features of language.

How to Gather Evidence:

- Review unit, chapter, and subunit overviews, as well as lessons, tasks and activities (scaffolds, call out boxes) for identification of language development supports²⁷ that are specific to the task and topic under study and provide access to grade-level work.
- Review teacher front matter, and teacher resources for guidance on small-group instruction and differentiation related to language development.

Examples of Evidence:

- Integration of writing across the content area with an emphasis on the value of writing as a tool for learning, thinking, and communication (e.g., writing for varied audience; writing like a mathematician).
- Lesson-level, embedded scaffolds that support students' academic reasoning and language skills (e.g., identifying cognates, sentence frames/starters, word banks, electronic dictionary or glossary).²⁸
- Guidance on grouping for discourse opportunities encourages students to use their oral language resources while they work collaboratively to develop content language and co-construct knowledge.
- Lesson-level use of multiple representations that provide students opportunities to discuss multiple meanings of math content, solutions, tasks, and activities (e.g., written representation, symbolic representation, visual representation, gesturing).

²⁶ Moschkovich, J. (2012). Mathematics, the Common Core, and language: Recommendations for mathematics instruction for ELs aligned with the Common Core. https://ul.stanford.edu/sites/default/files/resource/2021-12/02-JMoschkovich%20Math%20FINAL_bound%20with%20appendix.pdf

²⁷ Research-based strategies, scaffolds, and resources (e.g., sentence frames, visuals, word banks, and structured discussions) that help students develop mathematical understanding while strengthening their ability to communicate mathematically.

²⁸ CAST (2024). Universal Design for Learning Guidelines version 3.0, Representation, Language & Symbols, Consideration 2.3: Cultivate understanding and respect across languages and dialects. <https://udlguidelines.cast.org/representation/language-symbols/languages-dialects/>



Red Flags:

- Scaffolds attend to language development outside of the context of the task or topic (e.g., disconnected language mini-lesson using different content).
- Supports do not align with language proficiency levels or provide minimal access to the task or content.
- Grouping recommendations focus on static labels of perceived ability (e.g., “on grade-level”, “advanced”, “remedial”, etc.) and do not allow students to engage with a range of peers or to learn alongside students with a range of assets, language proficiencies, and experiences.

RELEVANT CONTEXTS

Instructional materials provide contextualized tasks and problems — and opportunities to contextualize tasks and problems — that incorporate students' everyday lives, families, and communities' ways of knowing, including their language and culture.²⁹

Key Definitions:

- **Contextualized tasks:** A math task or activity that is presented within a relatable real-life scenario, allowing students to connect the mathematical concepts to situations they can understand and potentially encounter in their daily lives.

How to Gather Evidence:

- Review tasks and contextualized problems across lessons for examples that affirm and center students' cultural backgrounds, experiences, and communities.

Examples of Evidence:

- Lessons include reflection prompts that encourage students to relate mathematical concepts to their personal experiences (e.g., "Think about a time you used this type of thinking in your daily life—how did it help you solve a problem?").
- Tasks allow students to choose from multiple contexts when applying math concepts (e.g., analyzing data related to local transportation, community health, or school demographics).
- Word problems and projects integrate themes that are age-relevant and engaging (e.g., middle school students modeling expenses for a school event, high school students analyzing the financial impact of college loans or career earnings).
- Math applications reflect students' realities, such as budgeting with culturally relevant foods, measuring distances using familiar locations, or analyzing statistics on media consumption and social trends.

Red Flags:

- Tasks that are devoid of contextualized relevance, absent connections to students community, culture and the assets they bring to the classroom.
- Contextualized tasks that are superficial or tokenistic, offering no meaningful connection to students' lived experiences or opportunities for deeper reflection and action.

²⁹ Celedón-Pattichis, S., Borden, L. L., Pape, S. J., Clements, D. H., Peters, S. A., Males, J. R., ... & Leonard, J. (2018). Asset-based approaches to equitable mathematics education research and practice. *Journal for Research in Mathematics Education*, 49(4), 373–389.

MATHEMATICAL DISCOURSE

Instructional materials are designed to allow for students to shape the mathematical discourse by specifying opportunities for students to listen to, share with, and build on peer mathematical thinking.

Key Definitions:

- None

How to Gather Evidence:

- Review unit and subunit overviews, lesson overviews and structures (e.g., callout boxes), and lessons for embedded discourse structures that guide students in sharing, listening to, and building on peer mathematical thinking.
- Within tasks and lessons, identify grouping structures (e.g., partner work, small groups, peer-led discussions) to determine how they promote mathematical discourse and collaborative learning.
- Review design features for protocols, and introductory lessons for explicit instruction on discussion norms and building a collaborative math learning community.

Examples of Evidence:

- Daily tasks and prompts that encourage students to explain their reasoning, critique peer strategies, or build on each other's ideas.
- Extended and supportive opportunities for discourse about grade-level tasks that include clear discussion structures, prompts, student roles, and supports for engaging (e.g., slides, protocols, anchor charts, student-facing materials).³⁰
- Teacher guidance on leveraging the five practices for facilitating student discourse (e.g., anticipate, monitor, select, sequence, and connect).³¹
- Teacher-facing resources with guidance on facilitating daily discourse, including sample questions, sentence stems, and feedback strategies.
- Examples of varied grouping structures (e.g., partners, small-groups, whole group, peer-led) in lesson plans that support equitable participation in mathematical discourse.
- Embedded discourse structures such as language and instructional routines (e.g., think-pair-share, stop-and-jot, daily routines to debrief, reflect, or synthesize learning through discussion).

³⁰ CAST (2024). Universal Design for Learning Guidelines version 3.0, Engagement, Sustaining Effort and Persistence, Consideration 8.3: Foster collaboration, interdependence, and collective learning. <https://udlguidelines.cast.org/engagement/effort-persistence/collaboration/>

³¹ Nabb, K., Hofacker, E. B., Ernie, K. T., & Ahrendt, S. (2018). Using the 5 practices in mathematics teaching. *The Mathematics Teacher*, 111(5), 366-373.

Red Flags:

- Discourse opportunities are limited to teacher-led discussions, with few or no opportunities for student-to-student dialogue.
- Little to no guidance on facilitating discourse daily.
- Discourse is only used as a tool to get students to the “right” answer or review steps and procedures.

PRACTICE OPPORTUNITIES AND RESOURCES

Instructional materials include well-designed, grade-level practice opportunities that focus on essential mathematics and align within the progression. These materials include all of the following elements:

- a. a variety of modes and meaningful contexts (e.g., games, puzzles, whiteboards, card sorts, interactive problem-solving);
- b. low floor, high ceiling — a flexible range of access and challenge that allows students to engage and practice across a spectrum of problems;
- c. purpose over quantity — intentional and clear connections to the current learning progression and involvement of students in reflection and self-assessment through the provision of solutions (calculations, representations, and/or writing) with reflection prompts to mark progress toward goals; and
- d. fluency — design that supports the deep connections between conceptual understanding and fluency.

Key Definitions:

- **Low floor, high ceiling:** A task or activity design that allows all students to access the mathematics at an entry point appropriate to their level of understanding (low floor), while also providing opportunities for deeper thinking, extension, or challenge (high ceiling) within the same task.

How to Gather Evidence:

- Identify tasks, activities, games, centers, and practice problems that include a variety of modes and contexts. Identify connections to learning progressions that provide in depth problems and tasks that include attention to fluency-building over time.
- Review teacher-facing materials for routines and structures that promote student reflection, self-regulation, and revision of their work.
- Review supplemental materials (e.g., enrichments, extensions, differentiation) for additional practice opportunities that build fluency, apply math concepts to contexts, or extend understanding.
- Review curriculum overview for approach to homework and additional practice resources.

Examples of Evidence:

- Tasks and routines that allow students to share solution strategies, revise thinking after peer discussions (e.g., returning to strategies, solution methods, or student work), and reflect on their learning process.
- Opportunities for students to self-select resources, methods, and representations to deepen understanding or demonstrate learning (e.g., orally, in writing, and using manipulatives, symbols, and multiple representations).
- Additional practice materials that allow for multiple entry points and align with grade-level standards while offering opportunities to apply concepts, further develop procedural skill and/or fluency, and extend math concepts beyond the lesson.
- Practice problems that spiral content from throughout the year, allowing for opportunities to develop procedural skill and fluency over time.

- A variety of routines, structures, and tasks that allow for whole group/small group/partnership collaborative tasks.
- Availability of additional practice materials (not just those provided for the lesson) that provide opportunities to further develop procedural skill and/or fluency, apply math concepts to contexts, and extend math concepts more deeply.

Red Flags:

- Practice tasks that are limited to rote procedures without opportunities for collaborative work, reflection, or choice in demonstrating learning.
- Supplemental materials that are either absent, misaligned with the lesson, or fail to deepen conceptual understanding alongside fluency development.
- Insufficient practice opportunities following a conceptual lesson sequence. (For example spending multiple lessons developing conceptual understanding of multi-digit division but moving forward without sufficient opportunities to apply and practice).

PROGRESS MONITORING

Instructional materials embed frequent opportunities for students to demonstrate understanding of grade-level mathematical concepts using their existing language resources.³² They also embed resources and frequent opportunities to monitor and respond to students' understanding of grade-level mathematics. Materials demonstrate how to diagnose critical student needs and draw clear connections to integrating supports and prioritizing instruction. These materials include all of the following elements:

- a. embedded and consistent formative assessment practices for mathematical content, mathematical literacy, and language learning;
- b. varied ways and multiple means of using formative data (including opportunities, beyond calculation alone, to explain, write, represent, self-reflect, and connect ideas) to demonstrate students' mathematical thinking and to make instructional decisions based on students' mathematical thinking; and
- c. regular monitoring of grade-level mathematics development.

Key Definitions:

- **Formative assessment practices:** Collaborative processes and data collection to understand students' learning and identify strengths and areas for improvement; used by teachers for instructional planning and by students to deepen their understanding.

How to Gather Evidence:

- Review scope and sequence, assessment guidance documents, unit overviews, and lessons for explicit opportunities for progress monitoring (e.g., exit tickets, assessments, collecting student work, observation checklists, etc.) that attend to the progression of mathematical content.
- Review units and lessons for diverse activities for students to demonstrate their learning. (e.g., discussion, quizzes, checks for understanding, collaborative projects).
- Identify all instructional assessments (e.g., diagnostic, formative, summative) for grade-level math proficiency over the year or course.
- Review teacher-facing resources for guidance in using student learning data to make informed instructional decisions that prioritize student needs and supports.
- Review daily lessons for opportunities to assess and respond to student understanding at the lesson level.
- Analyze chapter and unit level assessments for balance of question types (e.g., multiple select, simple calculation, constructed response, modeling, argumentation).

³² English Learners Success Forum. (n.d.). Math guidelines. Retrieved July 11, 2023, from www.elsuccessforum.org/mathguidelines

Examples of Evidence:

- Diagnostic tools such as beginning-of-unit pre-assessments and guidance on how to use the data to embed supports throughout the unit.
- Multiple opportunities within each lesson for students to show their thinking, skills, and knowledge (e.g., partner discussions, written representation, exit tickets, whiteboards, hand signals, etc.).³³
- Culminating tasks, projects, or performance tasks provide opportunities to demonstrate knowledge related to topics under study and are scaffolded throughout the unit.
- Assessment items align to the level(s) of rigor called for in the standards.
- Embedded and consistent formative assessment practices that allow for daily monitoring of grade-level mathematics development, mathematical literacy, and language learning.
- Varied ways and multiple means of using formative data (including opportunities, beyond calculation alone, to explain, write, represent, self-reflect, and connect ideas) to demonstrate students' mathematical thinking and to make instructional decisions based on students' mathematical thinking.
- **Grades K-2:**
 - Instructional materials provide structured teacher checklists to document students' mathematical thinking and strategies during hands-on activities, discussions, and play-based learning, ensuring ongoing informal assessment.
 - Materials include assessments where items are read aloud, allowing students to respond verbally or use manipulatives to demonstrate their mathematical understanding.

Red Flags:

- Assessments are primarily summative in nature (e.g., end-of-unit test, project or assignment) and do not provide ongoing evidence of student learning.
- Assessment items focus on an isolated skill that is not content covered in the unit or lesson's topic of study.
- Assessment scoring and response guidance suggests leveling students in ways that regularly restrict their access to grade-level content.
- Assessments isolate the assessment of language and content (e.g., assess language skills outside of the content knowledge students are learning about).

³³ CAST (2024). Universal Design for Learning Guidelines version 3.0, Representation, Building Knowledge, Consideration 3.4: Maximize transfer and generalization. <https://udlguidelines.cast.org/representation/building-knowledge/transfer-generalization/>

MEANINGFUL FEEDBACK

Instructional materials provide frequent opportunities and facilitation notes on how to provide meaningful feedback to advance mathematical understanding and language. These materials include all of the following elements:

- a. peer and teacher *cycles of feedback*, including communicating progress with affirming evidence of mathematical progress;
- b. normalization of mistake-making and affirmation of effort and growth;
- c. guidance for explicit, timely, informative, and accessible formative feedback to address partial solutions and alternative thinking in ways that allow learners to monitor their own progress effectively and to use that information to guide their own effort and practice without sacrificing their math confidence;
- d. focus among students on sense-making and/or metacognitive processes; and
- e. guidance on how and when to collect data, as well as how to respond to specific student strengths and needs.

Key Definitions:

- **Cycles of feedback:** Regularly repeated processes at key points in the learning experience to evaluate students' understandings and skills in ways that are iterative (continuous efforts to gather information, assess, reflect, identify growth areas, and refine practice).

How to Gather Evidence:

- Review units and lessons for identification of opportunities (e.g., questioning supports, sample dialogue, student work samples) for teacher- or peer-led feedback.
- Review lesson activities and assessments for processes that support revisions and improvements over time (e.g., opportunities for multiple drafts, peer feedback).
- Review units, lessons, and teacher-facing resources (e.g., assessment guidance, answer keys, sample work, and tasks) for content- and task-specific guidance that supports students to work through misconceptions and partial understandings by leveraging their assets.

Examples of Evidence:

- Lesson-embedded suggestions for:
 - When teachers should collect student work or collect in-the-moment data about student understandings (e.g., checklist observational notes).
 - Clear protocols or guidance for how to review students' work/thinking (e.g., what to look for; sample student responses).
 - Task- and content-specific guidance for providing feedback (e.g., checklist, conference template, sample oral feedback).
- Regular opportunities for students to review each other's work and clear ways to support student-to-student feedback (e.g., checklist, sentence starters, feedback protocol, models).
- Regular opportunities for students to revise work based on feedback.

- Guidance on how teachers can model and highlight students' efforts and growth (e.g., routines for end of class summaries or reflection prompt).
- Clear guidance about how teachers should prioritize feedback to students based on the information they collected (e.g., focusing on only the highest leverage piece(s) of feedback at a time).³⁴
- Suggestions for differentiated and flexible instruction including goals, instructional practices, and materials that connect to specific students' strengths and needs.
- Call-outs or teacher guidance on how to normalize mistakes (e.g., examples of how to leverage common misconceptions to further learning, professional readings on the importance of mistake-making in math).
- Examples of possible student misconceptions with guidance on how to respond at the lesson and unit level.

Red Flags:

- Feedback guidance is overly formulaic, not customized to specific student strengths/needs, or focused on a fixed notion of performance, compliance, or competition (e.g., relative performance).
- Feedback guidance only addresses student needs (e.g., does not highlight opportunities to identify and build on student strengths).
- Feedback guidance does not attend to content development or metalinguistic awareness (e.g., consciously attending to, reflecting on, and addressing the features of math content).

³⁴ CAST (2024). Universal Design for Learning Guidelines version 3.0, Engagement, Sustaining Effort & Persistence, Consideration 8.5: Offer action-oriented feedback. <https://udlguidelines.cast.org/engagement/effort-persistence/feedback/>

Educator Supports

DOMAIN 1: EDUCATOR KNOWLEDGE

EXAMINATION OF SELF

Instructional materials support teachers in examining their identities, biases, and belief systems related to math instruction to better understand how these factors might influence instructional choices and the lens through which they interpret student thinking. These materials may include reflection prompts or embedded professional learning.

Key Definitions:

- **Biases:** The conscious or unconscious beliefs, assumptions, or attitudes that teachers hold; they are shaped by their own identities and experiences and can influence how they interpret student thinking and behavior, how they approach instruction, how they make instructional decisions, and how they respond to students

How to Gather Evidence:

- Review teacher-facing guidance, professional learning supports, and resources for inclusion of reflection prompts to explore self-identities, biases, or beliefs that may impact instructional decisions.
- Identify embedded professional learning opportunities (e.g., model lessons, annotated lesson plan, suggested professional reading) to guide teachers to reflect on their interpretation of student thinking.

Examples of Evidence:

- Prompts or reflection activities that invite teachers to identify and reflect on the way their identities, experiences, and knowledge impact how they view students and their thinking/work.
- Sample educator thinking or educator case studies that illustrate how beliefs impact instruction.
- Research summaries or prompts that interrupt bias about the inherent capabilities of multilingual learners or students with diverse learning needs.
- Prompts that encourage educators to explore their relationship with their own mathematics education, their own math identities, or their experiences with math anxiety.

Red Flags:

- No acknowledgement of users of the instructional materials.
- Provided content to address the support of students works from a frame of addressing deficits without acknowledging that all students have assets they bring to any mathematical task.

STUDENTS' LINGUISTIC AND CULTURAL ASSETS

Instructional materials support educators to leverage students' linguistic and cultural assets, approaching these assets with a disposition of curiosity and appreciation. These materials include prompts for educators to learn about and integrate the knowledge, strengths, and resources of students, families, and the community — especially those who have been historically marginalized.³⁵ This includes connecting to and bringing in math topics and ideas from the backgrounds of students, drawing from students' home and everyday language to learn mathematics, and building and strengthening relationships that elicit and center these assets to bridge and propel relevance of learning.³⁶

Key Definitions:

- **Linguistic and cultural assets:** The diverse language skills, cultural backgrounds, and lived experiences that students bring to the classroom.

How to Gather Evidence:

- Review program- and grade-level educator-facing materials for prompts, activities, or protocols that address student and community assets.
- Identify additional professional and collaborative learning opportunities (e.g., interactive modules, instructional frameworks, videos, webinars) for teachers to build asset-based language and community engagement strategies.
- Review take-home activities, family letters, or suggested discussions that connect mathematical learning to students' home lives and invite family and community participation.

Examples of Evidence:

- Student surveys or activities that help teachers learn about student linguistic and cultural assets.
- Family letters, surveys, or suggestions for family and community partnerships related to the topic of study.
- Professional reading at the unit and topic level, lesson callouts on ways that mathematical concepts are taught or represented in other cultures.

Red Flags:

- The research-based “why” behind instructional practices to support language development is not explicitly named.
- Educative materials silo educators' understanding of the development of language, content, and literacy skills (e.g., discuss vocabulary development outside of the content that students are learning).

³⁵ Aguirre, J., Mayfield-Ingram, K., & Martin, D. B. (2013). The impact of identity in K-8 mathematics learning and teaching: Rethinking equity-based practices. National Council of Teachers of Mathematics.

³⁶ Moschkovich, J. (2013). Principles and guidelines for equitable mathematics teaching practices and materials for English language learners. *Journal of Urban Mathematics Education*, 6(1), 45–57.

SUPPORTING LANGUAGE DEVELOPMENT FOR ALL LEARNERS

Instructional materials build educators' understanding of research-based practices to support language development for all learners, especially for multilingual learners and students with diverse learning needs, including (all of the following):

- a. developing explicit language objectives for communication about mathematics;³⁷
- b. building knowledge of students' language development and language development standards, as connected to the mathematics of the lesson or unit;
- c. enacting mathematical language routines to foster mathematical discourse and communication amongst students;
- d. providing examples of sample student responses, in the context of actual mathematics tasks, with a range of language proficiency;³⁸
- e. suggestions of ways to capture student progress from everyday language to language for more formal academic and mathematical purposes; and
- f. guidance on what to look for, listen for, questions to ask, and/or feedback to give when supporting multilingual learners.

Key Definitions:

- **Mathematical language routines:** Structured, research-based instructional strategies that support students in developing their mathematical reasoning, communication, and understanding by focusing on different aspects of language use (e.g., support sense-making, optimize output, cultivate conversation, maximize linguistic and cognitive meta-awareness).³⁹

How to Gather Evidence:

- Review program- and grade-level educator-facing materials for language development through prompts, activities, or additional professional learning (e.g., model lessons, instructional frameworks, interactive modules).
- Identify teacher guidance and materials that support the development of academic vocabulary, increasing comprehension, building background knowledge, and making cross-linguistic connections.⁴⁰

³⁷ Gottlieb, M., & Ernst-Slavit, G. (2014). Academic language in diverse classrooms: Definitions and contexts. Corwin.

³⁸ English Learners Success Forum. (n.d.). Math guideline 13. Retrieved April 1, 2025, from <https://www.elsuccessforum.org/math-guidelines/math-area-of-focus-5>

³⁹ Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). Principles for the Design of Mathematics Curricula: Promoting Language and Content Development. Retrieved from Stanford University, UL/SCALE website: <http://ell.stanford.edu/content/mathematics-resources-additional-resources>

⁴⁰ Identifying and exploring the ways that languages are the same and different (e.g., sound, spelling, vocabulary, syntax).



Examples of Evidence:

- Callouts, readings, instructional videos, Professional Learning Community agendas/activities, or other professional learning opportunities with a focus on how language develops, as connected to the mathematics of the lesson or unit.
- Support for educators weaves together the development of language, content, and mathematics skills (e.g., identify essential content knowledge and language skill for each lesson).
- Checklists or rubrics to capture student progress from everyday language to language for more formal academic and mathematical purposes, as well as guidance on what to look for, listen for, questions to ask, and/or feedback to give when supporting multilingual learners.
- Student scaffolds for comprehension and communication, such as sentence frames, visual aids, as well as probing questions.
- Resources for educators that support the development of cross-linguistic connections (e.g., cross-linguistic glossaries, comparisons of linguistic structures, alphabet or sound/spelling charts in different languages) and engage teachers to learn and develop curiosity about students' linguistic assets.
- Explicit language objectives for communication about mathematics or guidance for developing and aligning objectives to mathematics content.
- Mathematical language routines to foster mathematical discourse and communication amongst students with guidance on when and how to use them.
- Examples of sample student responses, in the context of actual mathematics tasks, with a range of language proficiency.

Red Flags:

- The research-based “why” behind instructional practices to support language development is not explicitly named.

SUPPORTING MATHEMATICAL DEVELOPMENT

Instructional materials deepen educators' mathematical knowledge for teaching through building educators' understanding of research-based practices to support routines for reasoning, inquiry-based approaches, and structures that develop and affirm positive math mindsets during the process of supporting all students in understanding grade-level mathematics. These materials include all of the following elements:

- a. teacher guidance on multiple math strategies and the ways in which those approaches represent different, but equally valid, conceptions of the same mathematical idea(s); and
- b. guidance on what to look for, what to listen for, questions to ask, and/or feedback to give so that mathematical inquiry and reasoning is student led.

Key Definitions:

- **Positive math mindset:** A belief that mathematical ability grows through effort, persistence, and reasoning, fostering confidence, curiosity, and a willingness to engage in problem-solving and learning from mistakes.

How to Gather Evidence:

- Review program- and grade-level educator-facing materials for attention to mathematical development that relate to key ideas in the indicator through prompts, activities, or additional professional learning.
- Identify additional professional and collaborative learning opportunities (e.g., interactive modules, instructional frameworks, videos, webinars) for teachers to build and deepen understanding of research-based practices that support student reasoning and inquiry.

Examples of Evidence:

- Callouts, readings, instructional videos, Professional Learning Community agendas/activities, or other professional learning opportunities embedded within instructional materials with a focus on understanding research-based practice.
- Multiple visual representations, numerical methods, and algebraic reasoning to illustrate varied math strategies and approaches.
- Prompts, discussion questions, sample dialogues, or potential feedback to guide teachers in fostering student-led mathematical inquiry.
- Teacher guidance on anticipating common student responses—both correct and incorrect—and insights into the underlying mathematical reasoning behind them. This includes examples of student strategies, potential misconceptions, and recommended teacher moves to respond in ways that deepen student understanding.
- Instructional routines and prompts to guide metacognitive processes embedded at the lesson level to encourage student sense-making, reasoning, reflection and agency, with guidance on when and how to use them.
- Professional readings on key topics such as the mathematical standards, instructional shifts, and the Standards for Mathematical Practice (SMPs) to deepen educators' understanding of learning progressions.

- Instructional materials include explicit connections to prior and future learning, helping educators contextualize current grade-level concepts within a multi-year progression. (e.g., In Grade 3, students understand concepts of area and relate area to multiplication and to addition, and then in Grade 5, students use area models to understand multiplication with decimal numbers.)

Red Flags:

- Instructional materials present only one method for solving problems without acknowledging alternative, equally valid approaches, limiting teachers' ability to support diverse ways of thinking and reasoning.

GUIDANCE FOR MATHEMATICAL DISCOURSE

Instructional materials are designed to foster educator facilitation of mathematical discourse shaped by students through specifying opportunities for students to listen to, share with, and build on peer mathematical thinking. These materials include guidance on structuring student activities that have all of the following elements:

- a. sharing their own mathematical thinking with their peers;
- b. engaging with their peers' mathematical thinking;
- c. reflecting on and articulating their own understanding of their peers' mathematical perspectives;
- d. building on and extending their peers' mathematical ideas; and
- e. providing feedback to their peers on their mathematical reasoning.

Key Definitions:

- None

How to Gather Evidence:

- Review unit and lesson preparation materials for guidance on facilitation of mathematical discourse.
- Review lessons for opportunities for students to engage with (e.g., building on, adding to and restating shared ideas, questioning) one another's mathematical thinking during activities.

Examples of Evidence:

- Guidance for structuring activities (e.g., Think-Pair-Share, gallery walks, small group discussions) that enable students to share their mathematical thinking with their peers, as well as guidance on how to leverage activities to foster mathematical discourse.
- Sentence frames, discussion protocols, or rubrics to support students in explaining their thinking and understanding their peers' thinking.
- Guidance on how to purposefully sequence student contributions during discussions to highlight a progression of mathematical ideas. This includes sample discussion flows, suggested teacher prompts to connect ideas, and strategies for ensuring all student voices contribute to a collective understanding.
- Sample teacher prompts and questioning strategies to guide students in engaging with their peers' mathematical thinking, including building on and extending their peers' mathematical ideas.

Red Flags:

- Materials do not provide specific strategies or examples for facilitating mathematical discourse, such as prompts, sentence stems, or structured activities that encourage students to share, engage with, and build on each other's ideas.
- Opportunities for student discussion are limited to surface-level sharing (e.g., "turn and talk") without structured protocols, follow-ups, such as reflecting on peers' reasoning, offering feedback, or connecting ideas to broader mathematical concepts.

COLLECTIVIST APPROACH

Instructional materials provide teacher guidance that counters traditional math structures of individualism and competition by structuring the doing of mathematics through collaboration.

Key Definitions:

- **Competition (in math):** A classroom dynamic where students are pitted against one another to solve problems quickly or outperform peers, often prioritizing speed and correctness over deep understanding and collaboration.

How to Gather Evidence:

- Review lesson details, materials, and resources for guidance on fostering a collectivist approach to mathematics, balancing independent with partner and group work.

Examples of Evidence:

- Callouts, readings, instructional videos, Professional Learning Community agendas/activities, or other professional learning opportunities embedded within instructional materials that create opportunities to explore beliefs about mathematics teaching and learning.
- Activities and prompts that support communalism,⁴¹ such as group problem-solving projects and peer-to-peer interactions.

Red Flags:

- Materials over-emphasize independence, competition, or a single, "correct" way of engaging in math.

⁴¹ Ortiz, N., & Morton, T. (2022, May). Empowering black mathematics students through a framework of communalism and collective black identity. *Journal of Urban Mathematics Education*, 15(1), 54-77.

DOMAIN 2: USABILITY

DESIGN AND FUNCTIONALITY

Instructional materials are designed to support ease of student and teacher use. These materials include all of the following elements:

- a. scalability and accessibility and the ability for the curriculum to be disseminated in a way that ensures equitable student, teacher, and community access;
- b. visually appealing design with an organized and logical format;
- c. appropriate pacing;
- d. clear and concise educator-facing guidance that enables educators to prepare lessons in a timely manner; and
- e. a variety of ways to engage with the content, including leveraging current technology and tools (e.g., online graphing calculators, digital manipulatives).

Key Definitions:

- **Scalability:** The ability of instructional materials to be effectively implemented across different classroom sizes, school settings, and student populations while maintaining quality and usability.
- **Appropriate pacing:** A structured progression of lessons that ensures sufficient time for concept development, practice, and application, allowing students to engage deeply with grade-level mathematics without feeling rushed or stalled.

How to Gather Evidence:

- Review scope and sequence documents, for provided pacing, instructional components, and methods of accessing materials.
- Review units and lessons to determine how design, material organization, and language contribute to or detract from usability and functionality.
- Identify features that support students with diverse learning needs (e.g., alternative formats, compatibility with assistive tools).
- Assess overall materials for layout, typography, and use of images that are user-friendly for students and teachers.
- Identify technology features for delivery of content or engagement that enhance learning experiences (e.g., slide decks, screen images, digital graphing programs, virtual manipulatives).

Examples of Evidence:

- Student- and teacher-facing resources are available in digital formats that can support text-to-speech or other assistive technology.
- Digital and print resources are clearly marked and organized (e.g., clearly organized digital interface; use of color-coding or visuals; consistent labeling) with opportunities to use online graphing calculators or digital manipulatives where appropriate.

- Materials for one lesson or instructional experience are easy to access, ideally in one place.
- Estimated pacing of activities is adequate (e.g., suggested timing is considered for each individual task; additional time built into unit pacing for teacher-directed adjustments; time built into scope and sequence for necessary teacher adaptations).
- Clear guidance about core instructional components and supplemental components.
- Lessons orient educators to the most important information (e.g., overview of lesson structure alongside needed materials and preparation).
- Teacher guidance is written in an easy-to-digest way that effectively and efficiently describes lesson activities (e.g., teachers can read and prepare to teach a lesson in 45 minutes or less).
- The program includes clearly labeled, durable storage containers for manipulatives, making it easy for teachers to store, organize, and distribute materials efficiently during instruction.
- Instructional materials provide pre-made, ready-to-use resources such as printed card sets, game pieces, and manipulatives that reduce the need for extensive teacher preparation (e.g., printing, cutting, laminating, and storing).

Red Flags:

- Student- or teacher-facing materials are only available in non-editable format, creating an unnecessary burden for educators to adapt to their context.
- Print-only resources do not allow for assistive technology.
- Volume of materials or additional optional components with unclear guidance what is core instruction (and/or variance in the quality of volume of materials).
- Instructional materials are spread across multiple separate resources, requiring educators to reference several books or platforms simultaneously, making lesson preparation and implementation inefficient and difficult to manage.
- Teacher guidance exists solely in sidebars, call out boxes, one-offs or front-matter without additional lesson or task suggestions for effective implementation.

ADAPTABILITY FOR CONTEXT

Instructional materials contain materials and/or meaningful suggestions for how to adapt for district, school, and/or classroom context. These materials may include varied selections for topics under study; flexibility to modify tasks to connect to local resources, organizations, or issues; or varied pacing suggestions based on number of school days or minutes of instruction.

Key Definitions:

- None

How to Gather Evidence:

- Review the table of contents, scope and sequence documents, instructional and assessment days, for attention to varied instructional context restraints (e.g., school days and instructional minutes) and options for choice-based or adaptable units of study.
- Review lesson guidance, suggestions, materials, and resources for adaptability support.
- Identify and determine whether the presence of all lessons and adaptable elements are additional, optional, or required.

Examples of Evidence:

- Guidance or adapted lesson materials for varied contexts (e.g., options for a 90- or 120-minute instructional block; must do/may do instructional components; extensions).
- Materials offer a selection of units or arcs of learning that schools and/or teachers could select from (e.g., selecting from two sample units on sustainability—food sustainability and urban renewable energy; a range of research project materials on the same topic).
- Invitations within each grade level to attend to local context through task or topic. (e.g., suggestions on how to use data from a local farm in a statistics unit).

Red Flags:

- Instructional materials are strictly paced for daily instructional minutes and school days that are not feasible for local contexts.

PROGRAM COHERENCE

Core instructional materials guide the use of additional supplemental mathematics materials (e.g., interventional materials) in content and approach. Use of supplemental mathematics materials supports students in accessing the grade-level mathematics content that is concurrently happening in core instruction.

Key Definitions:

- **Core instructional materials:** Main instructional materials used to engage students in daily literacy instruction.
- **Supplemental mathematics materials:** Additional materials designed to support students outside of the core instructional block.

How to Gather Evidence:

- Determine the presence of supplementary materials within and across grade levels, including how supplemental materials are integrated into core instruction (e.g., remediation, extension materials outside of core instruction).
- Review supplemental materials to determine instructional strategies and approaches that are aligned to core content and research-based instructional practices.
- Review homework and optional practice resources for alignment around models, strategies, and language.
- Review assessments for alignment with instructional strategies and language.

Examples of Evidence:

- Included supplementary or intervention materials attend to addressing students' strengths and needs in ways that systematically leverage and build their foundational concepts, fluency with grade-level skills, and language and reasoning skills.
- Assessment materials in core instruction provide clear guidance and connections to supplemental materials.
- Assessment design and tasks align with instruction.
- Guidance on when and how to use supplementary materials so that they are in alignment with core instructional materials (e.g., previewing topics vs. re-teaching/reviewing topics).

Red Flags:

- Included supplementary or intervention materials are not research-based (e.g., focus on procedural knowledge absent of or with surface-level development of conceptual understanding).
- Homework and/or optional practice resources are not aligned to core instructional models.

Additional Resources

GRADE-LEVEL AND STANDARDS ALIGNED INSTRUCTION:

- [Maryland College and Career Ready Standards for Mathematics](#), Maryland State Department of Education
- [Progression Documents](#), Student Achievement Partners
- [Guidelines for Assessment and Instruction in Mathematical Modeling Education](#), Consortium for Mathematics and its Application and Society for Industrial and Applied Mathematics

DESIGNED TO AFFIRM STUDENTS:

- [Culturally Responsive-Sustaining Education](#), New York State Education Department
- [Supporting Culturally Responsive Teaching Practices through Math Curricula](#), Mathematica, Inc
- [Guidelines for Math Instructional Materials](#), English Language Success Forum
- [Principles for the Design of Mathematics Curricula: Promoting Language and Content Development](#), Stanford University

INSTRUCTIONAL DESIGN:

- [e2 \(Essential X Equitable\) Instructional Practice Suite](#), Student Achievement Partners
- [Universal Design for Learning Guidelines](#), CAST

EDUCATOR SUPPORTS:

- [Principles for the Design of Mathematics Curricula: Promoting Language and Content Development](#), Stanford University Graduate School of Education
- [Guidelines for Math Instructional Materials](#), English Language Success Forum