



Mathematics

High-Quality Instructional Materials Identification Framework

2024-2025

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Why Instructional Materials Matter for Maryland Students

The students of Maryland are a vibrant community of diverse learners, including a growing number of multilingual students and students from various racial and cultural backgrounds.¹ Instructional materials designed to best serve these students must facilitate enriching, culturally responsive, and language-affirming environments for all students.

Students deserve the opportunity to engage with rigorous content that builds a strong foundation for their educational journey and empowers them with essential learning skills. High-quality math instructional materials offer students engagement with worthy and complex tasks and learning experiences that foster critical thinking abilities and language development and amplify student voice and agency. Additionally, these materials prioritize the affirmation of students' cultural and linguistic identities, attending to inclusive learning communities that connect education to their real-world experiences and provide the support and skill to ensure that students with diverse learning needs thrive.

By aligning with college and career readiness standards and research-based approaches, high-quality instructional materials unlock and support knowledge-building that encourages active learning and leads to dynamic demonstrations of knowledge from students. Furthermore, these materials offer support for educators, equipping them with the necessary tools, content knowledge, pedagogical expertise, and research-based practices to effectively engage students and adapt to diverse community and school contexts. With this comprehensive approach, instructional materials in Maryland have the potential to create transformative learning environments that prepare students from kindergarten through graduation for a future of choice and opportunity.

¹ In 2022, Maryland's student population included 33% Black, 33% White, 22% Latinx, and 7% Asian students, as well as 12% English learners, 12% students with disabilities, and an increasing proportion who face economic challenges ([Maryland State Department of Education](#)).

Document Introduction

This framework serves as a valuable resource for educators and stakeholders across the education sector to identify key criteria in truly high-quality instructional materials. It outlines the essential elements of outstanding curricula and offers clear guidelines on the instructional shifts and educator supports needed to foster meaningful learning experiences for students. To deliver the world-class education that the [Blueprint for Maryland's Future envisions](#), educators and leaders can rely on this framework in service of identifying the research-based, high-quality materials that are necessary to provide students with rigorous instruction, nurture spaces that affirm their cultural and linguistic identities, and ensure their continued progress and success each year.

This framework is grounded in extensive research aimed at defining the content, instructional practice, and instructional design present in high-quality instructional materials. These research-based elements are central to the criteria within this framework and critical to support the kinds of learning experiences that Maryland students deserve.

Despite its strengths as a resource for identifying high-quality instructional materials, there are limitations for how this framework can be used. While the document presents crucial guidelines, it is NOT intended to be exhaustive in addressing all the elements of instructional materials and practices needed to create an equitable experience for students. Additionally, this document is NOT a rubric, meaning it does not provide a checklist or a scoring system for the evaluation of instructional materials. Instead, it offers guidance on the essential components of high-quality materials, encouraging educators to exercise professional judgment and adapt to their specific educational context.

It is also important for educators and leaders to recognize any and all humanizing considerations beyond the framework that may be necessary based on their unique students, classroom contexts, and school/district conditions in their review and selection of high-quality materials. Overall, this framework serves as a roadmap, empowering educators to select and use instructional materials that foster inclusivity, rigor, and relevance, ultimately resulting in enhanced learning outcomes for all students.

DOCUMENT ORGANIZATION

This document, intended for use when considering K–12 mathematics core instructional materials, is organized into four categories (Designed to Affirm Students, Grade-Level and Standards-Aligned, Instructional Design, and Educator Supports), with domains that highlight key criteria in each section.

While specific categories have been included for culturally responsive-sustaining pedagogy and language-affirming instruction, related considerations for affirming students are woven throughout the framework. Similarly, considerations for diverse learning needs and Universal Design for Learning have been embedded throughout to reflect the way that these practices must be interlaced in thinking about content, instructional practice, and support for educators.

A collection of research and scholarship used to inform this framework is included as an appendix.

Key Criteria for High-Quality Instructional Materials

DESIGNED TO AFFIRM STUDENTS

Affirming students creates opportunities for cultural and linguistic backgrounds to be an asset and a source of validation in the learning experience. In addition to a foundation of grade-level content, high-quality instructional materials must prioritize instructional practices that affirm students' cultural and linguistic backgrounds and support students with a range of diverse learning needs to thrive through mathematics. This support includes developing culturally responsive-sustaining learning communities that center who students are, use mathematics as a tool for civic engagement, and connect learning to the world outside the schoolhouse walls. Mathematics instruction must also intentionally affirm students' languages and language practices through a focus on building upon students' multilingualism, ensuring that texts support disciplinary language development, and designing language objectives that work in concert with content and mathematics learning. Through these instructional choices, materials have the potential to deepen mathematics learning, cultivate a sense of belonging, and develop students' mathematical identities — to see themselves as participants in mathematics.²

Key Criteria for Culturally Responsive-Sustaining Instruction³

- **Affirmation and Centering of Students:** Instructional materials affirm, engage, and center past and current knowledge of Black/African, Indigenous, Brown, and non-Western conceptions of math and highlight multilingualism and non-Western mathematicians and their discoveries. Instructional materials are designed to encourage students to anchor learning in individual experiences, backgrounds, and cultural knowledge to expand their mathematics knowledge and skills.
- **Mathematics as a Tool for Civic Engagement:** Instructional materials consistently include tasks that prompt students to apply and develop their civic engagement skills and examine social contexts and current events, using mathematics to question the world and the current status quo.
- **Real-World Connections and Relevant Data:** Instructional materials consistently connect with students' lives, their future goals, their communities, and the world and nurture ways to engage in their own communities and beyond³. These materials include all of the following elements:
 - a. use of mathematical concepts and tasks to connect to current events;
 - b. collaborative tasks and/or projects that involve real-world problem-solving through meaningful interactions with peers and their local communities;
 - c. structures (e.g., tasks, classroom activities, routines, assignments) to explore mathematical concepts from current events and data that are relevant to students'

² Aguirre, J., Mayfield-Ingram, K., & Martin, D. B. (2013). The impact of identity in K-8 mathematics learning and teaching: Rethinking equity-based practices. National Council of Teachers of Mathematics.

³ This conceptualization of culturally responsive-sustaining instruction is built on evidence from its predecessors — culturally relevant, responsive, and sustaining pedagogies. This scholarship underscores the importance of leveraging the diverse backgrounds of students as assets in the classroom that can and should be sustained through intentional instructional design. For more information about relevant scholarship, please see the citations section in the appendix.

lives and communities so that students see themselves in the tasks and understand how the tasks relate to their context and promote a sense of belonging;⁴ and

- d. teacher guidance to support students in developing mathematical skills and knowledge that are relevant to their academic and professional goals.

Key Criteria for Language Affirming Instruction

- **Multilingualism in Mathematics:** Instructional materials are deliberately designed to honor and build upon students' language(s) as an asset, encouraging students to use their linguistic repertoire to communicate with one another via reading, writing, speaking, and listening while engaging in mathematical learning. These materials include all of the following elements:
 - a. providing facilitation and engagement support for students to communicate as they do the math, make meaning, and collaboratively solve problems;
 - b. building mathematical language and content in English and home language(s), including use of social and academic vocabulary, through translanguaging so all students express themselves in a language they are comfortable with while working to learn mathematical content and meet language objectives in the target language;⁵ and
 - c. making cross-linguistic connections, including identifying and comparing similarities and differences between home language(s) and English (e.g., cognates) or registers and registers of instruction.
- **Language Objectives:** Instructional materials provide explicit alignment between language and content objectives to ensure that the language goals embedded within the standards are being attended to in every lesson. This includes language objectives for both expressive (writing and speaking) and receptive (listening and reading) communication that are aligned to the math learning goal.
- **Cognitively Demanding Mathematics:** Instructional materials provide cognitively demanding mathematics tasks that offer multiple research-based entry points and linguistic scaffolds to meet the needs of multilingual learners and students with diverse learning needs.
- **Reasoning in Multiple Ways:** Instructional materials include tasks that invite students to share their reasoning in multiple ways and guidance (e.g., annotations for teachers facilitating the tasks) about encouraging students to transverse between and among different representations (e.g., oral language and pictorial representations, written word and math tools).

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⁴ Tate, W. F. (1995). Returning to the root: A culturally relevant approach to mathematics pedagogy. *Theory Into Practice*, 34(3), 166–173.

⁵ García, O., Johnson, S. I., & Seltzer, K. (2017). The translanguaging classroom: Leveraging student bilingualism for learning. Caslon. For more, see Translanguaging Strategies, English Learner Success Forum.

GRADE-LEVEL AND STANDARDS ALIGNED

Grade-level, standards-aligned content serves as a necessary foundation for equitable student experiences in the classroom. Engaging with this rigorous content from kindergarten through graduation sets students on a path to empowered lives, and instructional materials must be designed so that all students have access to this essential work. This includes ensuring all students are empowered by secure engagement with the most important and applicable mathematics of each grade or course; are positioned as mathematical leaders and doers in classrooms; leverage high-quality questions and tasks to practice and tune Standards of Mathematical Practice with content standards, develop their conceptual understanding, procedural skill and fluency, and application; and develop language along with mathematical content knowledge.

Key Criteria for Focus on Essential Mathematics

- **Essential Mathematics:** Instructional materials prioritize the most important and applicable concepts, knowledge, and mathematical skills. These materials include all of the following elements:
 - a. a focus on the major work of the grade (K–8)⁶ and Essential Concepts from *Catalyzing Change in High School Mathematics*⁷ (see appendix); and
 - b. guidance for students and teachers to use the materials as designed and spend most of their time focused on the essential mathematics of the grade/course.

Key Criteria for Coherence

- **Consistent Progressions:** Instructional materials are consistent with the progressions in college and career-ready standards. These materials include all of the following elements:
 - a. content progressions based on the grade-by-grade and course-by-course progressions in college and career-ready standards;
 - b. extensive work for all students in grade-level or high school course-level problems; and
 - c. grade-level or high school course-level concepts that are explicitly related to prior knowledge from earlier grades or courses.
- **Coherent Connections:** Instructional materials foster coherence through connections within a single grade, or course, where appropriate and where required by college and career-ready standards. These materials include all of the following elements:
 - a. supporting content to further engage students in the major work of the grade in K–8 and supporting content to further engage students in Essential Concepts in high school; and

⁶ National Governors Association, Council of Chief State School Officers, Achieve, & National Association of State Boards of Education. (2013, April 9). *K–8 publishers' criteria for the Common Core State Standards for Mathematics*. https://www.thecorestandards.org/wp-content/uploads/Math_Publishers_Criteria_K-8_Spring_2013_FINAL1.pdf

⁷ National Council of Teachers of Mathematics. (2018). *Catalyzing change in high school mathematics: Initiating critical conversations*.

- b. preservation of the focus, coherence, and rigor of college and career-ready standards even when targeting specific objectives.

Key Criteria for Rigor and Balance

- **Rigor and Balance:** Instructional materials reflect the aspect(s) of rigor — conceptual understanding, procedural skill and fluency, and/or application — called for by the standards.⁸ These materials include all of the following elements:
 - a. development of students’ conceptual understanding of key mathematical concepts, especially where called for in specific content standards or cluster headings;
 - b. attention throughout the year to procedural skill and required fluencies of each grade-level⁹; and
 - c. sufficient time for teachers and students to use the materials as designed and work with applications that engage students in problem-solving.

Key Criteria for Mathematical Practices

- **Practice-Content Connections:** Instructional materials meaningfully integrate Standards for Mathematical Practice with content standards and attend to the full meaning of each practice standard in tasks and problems.¹⁰
- **Emphasis on Mathematical Reasoning:** Instructional materials support the standards’ emphasis on mathematical reasoning through indicating and providing guidance about the opportunities for discourse, communication, problem-solving, and modeling.

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⁸ The three aspects of rigor are not always separate in materials. (Conceptual understanding and fluency go hand in hand, fluency can be practiced in the context of applications, and brief applications can build conceptual understanding.) Nor are the three aspects of rigor always together in materials. (Fluency requires dedicated practice. Rich applications cannot always be shoehorned into the mathematics topic of the day. And conceptual understanding will not always come along unless explicitly taught.)

⁹ <https://achievethecore.org/category/774/mathematics-focus-by-grade-level>

¹⁰ National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). Common Core State Standards for Mathematics. https://learning.ccsso.org/wp-content/uploads/2022/11/Math_Standards1.pdf

INSTRUCTIONAL DESIGN

Instructional materials must attend to research-based instructional practices that support meaningful engagement for all students to be deemed high quality. It is through this intentional design that instructional materials contribute to learning communities in which students unlock knowledge, engage with tasks and peers as mathematicians, regularly demonstrate their learning, and experience joy in the math classroom. This type of learning community builds students' mathematical identities, allowing students to see themselves and their peers as mathematical experts, thinkers, and doers in the classroom.

Key Criteria for Student Agency

- **Metacognitive Processes:** Instructional materials develop students' metacognitive skills to promote understanding of math concepts by directly teaching and supporting students to monitor understanding and progress over time. These materials include all of the following elements:
 - a. setting goals, self-monitoring growth, and reflecting on the impact of students' choices and ongoing development as mathematical doers, critical thinkers, and communicators;
 - b. providing explicit practices to develop students' metalinguistic awareness around how language works in mathematics, language use, and choices connected to mathematical ideas;
 - c. modeling and developing strategies that support students in making their thinking visible through speaking, writing, or drawing their developing understanding; and
 - d. supporting students with diverse learning needs in developing metacognitive strategies.
- **Choice:** Instructional materials prompt teachers to provide students ample time to explore math concepts, during which students are given opportunities to make choices about how to spend time, whom to spend it with, and what materials are used.
- **Multiple Entry Points to Complex Tasks:** Instructional materials include tasks that are complex, with multiple entry points (e.g., allow for multiple solution strategies, encourage use of multiple representations) that promote collaboration and different ways of thinking and explaining.
- **Authentic Engagement as a Mathematician:** Instructional materials promote productive struggle and the mathematical modeling process through quality math tasks that are sequenced to build conceptual understanding and procedural skill and fluency, prioritize inquiry, provide opportunities to take risks, allow for rough draft thinking and multiple approaches, invite the use of math tools, and use mistakes for learning so that students engage in collaborative learning.
- **Collaborative Learning:** Instructional materials engage all students in collaborative learning through a variety of research-based routines, structures, and tasks that allow for whole-group, small-group, and independent thinking. Materials explicitly plan for students to demonstrate their curiosity and share their tentative thinking; ask questions; and adjust their understanding by listening to and building on one another's shared ideas.

Key Criteria for Monitoring Progress and Supporting Students

- **Supports and Scaffolds¹¹:** Instructional materials are designed to support a variety of student strengths and diverse learning needs in ways that are supported by research and maintain attention to grade-level content alongside practice standards. These materials include all of the following elements:
 - a. scaffolds and supports that are designed based on mathematical learning progressions and the coherence of math concepts across and within grades and courses;
 - b. guidance on identifying scaffolds and appropriate supports that build on students' mathematical thinking, ideas, and experiences; and
 - c. content-specific guidance on identifying and addressing potential individual student needs so that supports, scaffolds, and extensions can be effectively differentiated, including adjustments to content, process, or product.
- **Simultaneous Mathematical Meaning-Making and Language Development:** Instructional materials include intentional language learning opportunities alongside appropriate, research-based supports for multilingual learners and students with diverse learning needs to develop mathematical meaning-making and language simultaneously. Materials include questions for students to raise metalinguistic awareness of how language works in math and integrate language standards alongside mathematical content standards.¹²
- **Relevant Contexts:** Instructional materials provide contextualized tasks and problems — and opportunities to contextualize tasks and problems — that incorporate students' everyday lives, families, and communities' ways of knowing, including their language and culture.¹³
- **Mathematical Discourse:** Instructional materials are designed to allow for students to shape the mathematical discourse by specifying opportunities for students to listen to, share with, and build on peer mathematical thinking.
- **Practice Opportunities and Resources:** Instructional materials include well-designed, grade-level practice opportunities that focus on essential mathematics and align within the progression. These materials include all of the following elements:
 - a. a variety of modes and meaningful contexts (e.g., games, puzzles, whiteboards, card sorts, interactive problem-solving);
 - b. low floor, high ceiling — a flexible range of access and challenge that allows students to engage and practice across a spectrum of problems;

¹¹ Thoughtfully designed questions and tasks that provide access to grade-level, culturally responsive-sustaining, and language-affirming experiences for students are one form of support for students and are addressed in other sections of this framework.

¹² Moschkovich, J. (2012). Mathematics, the Common Core, and language: Recommendations for mathematics instruction for ELs aligned with the Common Core. https://ul.stanford.edu/sites/default/files/resource/2021-12/02-JMoschkovich%20Math%20FINAL_bound%20with%20appendix.pdf

¹³ Celedón-Pattichis, S., Borden, L. L., Pape, S. J., Clements, D. H., Peters, S. A., Males, J. R., ... & Leonard, J. (2018). Asset-based approaches to equitable mathematics education research and practice. *Journal for Research in Mathematics Education*, 49(4), 373–389.

- c. purpose over quantity — intentional and clear connections to the current learning progression and involvement of students in reflection and self-assessment through the provision of solutions (calculations, representations, and/or writing) with reflection prompts to mark progress toward goals; and
 - d. fluency — design that supports the deep connections between conceptual understanding and fluency.
- **Progress Monitoring:** Instructional materials embed frequent opportunities for students to demonstrate understanding of grade-level mathematical concepts using their existing language resources.¹⁴ They also embed resources and frequent opportunities to monitor and respond to students' understanding of grade-level mathematics. Materials demonstrate how to diagnose critical student needs and draw clear connections to integrating supports and prioritizing instruction. These materials include all of the following elements:
 - a. embedded and consistent formative assessment practices for mathematical content, mathematical literacy, and language learning;
 - b. varied ways and multiple means of using formative data (including opportunities, beyond calculation alone, to explain, write, represent, self-reflect, and connect ideas) to demonstrate students' mathematical thinking and to make instructional decisions based on students' mathematical thinking; and
 - c. regular monitoring of grade-level mathematics development.
 - **Meaningful Feedback:** Instructional materials provide frequent opportunities and facilitation notes on how to provide meaningful feedback to advance mathematical understanding and language. These materials include all of the following elements:
 - a. peer and teacher cycles of feedback, including communicating progress with affirming evidence of mathematical progress;
 - b. normalization of mistake-making and affirmation of effort and growth;
 - c. guidance for explicit, timely, informative, and accessible formative feedback to address partial solutions and alternative thinking in ways that allow learners to monitor their own progress effectively and to use that information to guide their own effort and practice without sacrificing their math confidence;
 - d. focus among students on sense-making and/or metacognitive processes; and
 - e. guidance on how and when to collect data, as well as how to respond to specific student strengths and needs.

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¹⁴ English Learners Success Forum. (n.d.). Math guidelines. Retrieved July 11, 2023, from www.elsuccessforum.org/math-guidelines

EDUCATOR SUPPORTS

To promote facilitation of meaningful learning experiences for all students, instructional materials ensure effective support for educators. Throughout the instructional materials, explicit tools and resources focus on enhancing educators' depth of mathematical knowledge for teaching, using pedagogical content knowledge in planning for instruction, and practicing responsive teaching to build on or extend students' mathematical thinking. These tools and resources also encourage reflective practices among educators, including the examination of their own identities as well as identifying places where teacher actions may contribute to building positive mathematical identities in their students. In addition, resources are thoughtfully designed for ease of use and fit to community context.

Key Criteria for Educator Knowledge

- **Examination of Self:** Instructional materials support teachers in examining their own identities, biases, and belief systems to help them understand how these factors might influence instructional choices and the lens through which they interpret student thinking. These materials may include reflection prompts, examples of educator thinking, or embedded professional learning.
- **Students' Linguistic and Cultural Assets:** Instructional materials support educators to leverage students' linguistic and cultural assets, approaching these assets with a disposition of curiosity and appreciation. These materials include prompts for educators to learn about and integrate the knowledge, strengths, and resources of students, families, and the community — especially those who have been historically marginalized.¹⁵ This includes connecting to and bringing in math topics and ideas from the backgrounds of students, drawing from students' home and everyday language to learn mathematics, and building and strengthening relationships that elicit and center these assets to bridge and propel relevance of learning.¹⁶
- **Supporting Language Development for all Learners:** Instructional materials build educators' understanding of research-based practices to support language development for all learners, especially for multilingual learners and students with diverse learning needs, including (all of the following):
 - a. developing explicit language objectives for communication about mathematics¹⁷;
 - b. building knowledge of students' language development and language development standards, as connected to the mathematics of the lesson or unit;
 - c. enacting math language routines¹⁸ to foster mathematical discourse and communication amongst students;

¹⁵ Aguirre, J., Mayfield-Ingram, K., & Martin, D. B. (2013). The impact of identity in K-8 mathematics learning and teaching: Rethinking equity-based practices. National Council of Teachers of Mathematics.

¹⁶ Moschkovich, J. (2013). Principles and guidelines for equitable mathematics teaching practices and materials for English language learners. *Journal of Urban Mathematics Education*, 6(1), 45–57.

¹⁷ Gottlieb, M., & Ernst-Slavit, G. (2014). *Academic language in diverse classrooms: Definitions and contexts*. Corwin.

¹⁸ English Learners Success Forum. (n.d.). Math guidelines. Retrieved July 11, 2023, from www.elsuccessforum.org/math-guidelines

- d. providing examples of sample student responses, in the context of actual mathematics tasks, with a range of language proficiency¹⁹;
 - e. suggestions of ways to capture student progress from everyday language to language for more formal academic and mathematical purposes; and
 - f. guidance on what to look for, listen for, questions to ask, and/or feedback to give when supporting multilingual learners.
- **Supporting Mathematical Development:** Instructional materials deepen educators' mathematical knowledge for teaching through building educators' understanding of research-based practices to support routines for reasoning, inquiry-based approaches, and structures that develop and affirm positive math mindsets during the process of supporting all students in understanding grade-level mathematics. These materials include all of the following elements:
 - a. teacher guidance on multiple math strategies and the ways in which those approaches represent different, but equally valid, conceptions of the same mathematical idea(s); and
 - b. guidance on what to look for, what to listen for, questions to ask, and/or feedback to give so that mathematical inquiry and reasoning is student led.
 - **Mathematical Discourse:** Instructional materials are designed to foster educator facilitation of mathematical discourse shaped by students through specifying opportunities for students to listen to, share with, and build on peer mathematical thinking. These materials include guidance on structuring student activities that have all of the following elements:
 - a. sharing their own mathematical thinking with their peers;
 - b. engaging with their peers' mathematical thinking;
 - c. reflecting on and articulating their own understanding of their peers' mathematical perspectives;
 - d. building on and extending their peers' mathematical ideas; and
 - e. providing feedback to their peers on their mathematical reasoning.
 - **Collectivist Approach:** Instructional materials provide teacher guidance that counters traditional math structures of individualism and competition by structuring the doing of mathematics through collaboration.

Key Criteria for Usability

- **Design and Functionality:** Instructional materials are designed to support ease of student and teacher use. These materials include all of the following elements:
 - a. scalability and accessibility and the ability for the curriculum to be disseminated in a way that ensures equitable student, teacher, and community access;

¹⁹ English Learners Success Forum. (n.d.). Math guidelines. Retrieved July 11, 2023, from www.elsuccessforum.org/math-guidelines

- b. visually appealing design with an organized and logical format;
 - c. appropriate pacing;
 - d. clear and concise educator-facing guidance that enables educators to prepare lessons in a timely manner; and
 - e. a variety of ways to engage with the content, including leveraging current technology and tools (e.g., online graphing calculators, digital manipulatives).
- **Adaptability for Context:** Instructional materials contain materials and/or meaningful suggestions for how to adapt for district, school, and/or classroom context. These materials may include varied selections for topics under study; flexibility to modify tasks to connect to local resources, organizations, or issues; or varied pacing suggestions based on number of school days or minutes of instruction.
 - **Program Coherence:** Core instructional materials guide the use of additional supplemental mathematics materials (e.g., interventional materials) in content and approach. Use of supplemental materials supports students in accessing the grade-level mathematics content that is concurrently happening in core instruction.

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Table 1. Progress to Algebra in Grades K–8

	K	1	2	3	4	5	6	7	8
Know number names and the count sequence		Represent and solve problems involving addition and subtraction		Represent & solve problems involving multiplication and division					
Count to tell the number of objects		Understand and apply properties of operations and the relationship between addition and subtraction		Understand properties of multiplication and the relationship between multiplication and division	Use the four operations with whole numbers to solve problems		Understand the place value system		
Compare numbers		Understand and apply properties of operations and the relationship between addition and subtraction	Represent and solve problems involving addition and subtraction	Multiply & divide within 100	Generalize place value understanding for multi-digit whole numbers		Perform operations with multi-digit whole numbers and decimals to hundredths	Apply and extend previous understandings of multiplication and division to divide fractions by fractions	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from		Add and subtract within 20	Add and subtract within 20	Solve problems involving the four operations, and identify & explain patterns in arithmetic	Use place value understanding and properties of operations to perform multi-digit arithmetic		Use equivalent fractions as a strategy to add and subtract fractions	Apply and extend previous understandings of numbers to the system of rational numbers	Analyze proportional relationship and use them to solve real-world and mathematical problems
Work with numbers 11–19 to gain foundations for place value		Work with addition and subtraction equations	Use place value understanding and properties of operations to add and subtract	Develop understanding of fractions as numbers	Extend understanding of fraction equivalence and ordering		Apply and extend previous understandings of multiplication and division to multiply and divide fractions	Apply and extend previous understandings of arithmetic to algebraic expressions	Use properties of operations to generate equivalent expressions
		Understand place value	Measure and estimate lengths in standard units	Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects	Build fractions from unit fractions by applying and extending previous understandings of operations		Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition	Reason about and solve one-variable equations and inequalities	Solve real-life and mathematical problems using numerical and algebraic expressions and equations
		Use place value understanding and properties of operations to add and subtract	Relate addition and subtraction to length	Geometric measurement: understand concepts of area and relate area to multiplication and to addition	Understand decimal notation for fractions, and compare decimal fractions		Graph points in the coordinate plane to solve real-world and mathematical problems*	Represent and analyze quantitative relationships between dependent and independent variables	Use functions to model relationships between quantities
		Measure lengths indirectly and by iterating length units							

*Indicates a cluster that is well thought of as part of a student’s progress to algebra, but that is currently not designated as Major by one or both of the assessment consortia in their draft materials. Apart from the asterisked exception, the clusters listed here are a subset of those designated as Major in both of the assessment consortia’s draft documents. ** Depends on similarity ideas from geometry to show that slope can be defined and then used to show that a linear equation has a graph which is a straight line and conversely.

K–8 Publishers’ Criteria for the Common Core State Standards for Mathematics, 9 Apr. 2013, achievethecore.org/content/upload/Math_Publishers_Criteria_K-8_Spring_2013_FINAL.pdf.

Research & Scholarship Supporting the Framework

A robust research and scholarship base underpins this framework. For more information about research-supported practice, see Student Achievement Partners' [Essential X Equitable Instructional Practice Framework™](#).

DESIGNED TO AFFIRM STUDENTS

Culturally Responsive-Sustaining

Gutstein, E., & Peterson, B. (Eds.). (2005). Rethinking mathematics: Teaching social justice by the numbers. Rethinking Schools.

Tate, W. F. (1995). Returning to the root: A culturally relevant approach to mathematics pedagogy. *Theory into practice*, 34(3), 166-173.

Language Affirming

Erath, K., Prediger, S., Quasthoff, U., & Heller, V. (2018). Discourse competence as important part of academic language proficiency in mathematics classrooms: The case of explaining to learn and learning to explain. *Educational Studies in Mathematics*, 99(2), 161-179.

García, O., Johnson, S. I., & Seltzer, K. (2017). *The Translanguaging Classroom: Leveraging Student Bilingualism for Learning*. Caslon. ; For more see https://assets-global.website-files.com/5b43fc97fcf4773f14ee92f3/5cca8e1dbfa8f118e41c578a_Translanguaging%20Strategies%20ELA.pdf

Gibbons, P. (2002). Scaffolding language, scaffolding learning: Teaching second language learners in the mainstream classroom. Portsmouth, NH: Heinemann.

Kazemi, E., & Hintz, A. (2014). *Intentional talk: How to structure and lead productive mathematical discussions*. Stenhouse Publishers.

Santamaria, L. J. (2009). Culturally responsive differentiated instruction: Narrowing gaps between best pedagogical practices benefiting all learners. *Teachers College Record*, 111(1), 214-247.

GRADE-LEVEL AND STANDARDS ALIGNED

Focus on Essential Mathematics

Adams, A.E., Karunakaran, M.S., Klosterman, P., Knott, L. & Ely, R. (2016, Nov). *Using Precise Mathematics Language To Engage Students In Mathematics Practices*. Full paper presentation accepted at the Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Tucson, AZ.: The University of Arizona. Retrieved from <https://files.eric.ed.gov/fulltext/ED583792.pdf>.

Catalyzing Change in High School Mathematics: Initiating Critical Conversations. The National Council of Teachers of Mathematics, Inc., 2018.

K-8 Publishers' Criteria for the Common Core State Standards for Mathematics, 9 Apr. 2013, achievethecore.org/content/upload/Math_Publishers_Criteria_K-8_Spring_2013_FINAL.pdf.

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