Maryland
STATE DEPARTMENT OF EDUCATION

Math High-Quality Instructional Materials Identification Framework

Office of Teaching and Learning

Why Instructional Materials Matter for Maryland Students

The students of Maryland are a vibrant community of diverse learners, including a growing number of multilingual students and students from various racial and cultural backgrounds.1 Instructional materials designed to best serve these students must facilitate enriching, culturally responsive, and language affirming environments for all students.

Students deserve the opportunity to engage with rigorous content that builds a strong foundation for their educational journey and empowers them with essential learning skills. High-quality literacy instructional materials offer students engagement with worthy and complex texts, tasks, and learning experiences which foster critical thinking abilities, language development, and amplify student voice and agency. Additionally, these materials prioritize the affirmation of students' cultural and linguistic identities, attending to inclusive learning communities that connect education to their real world experiences, and provide the support and skill to ensure students with diverse learning needs to thrive.

By aligning with College and Career Readiness standards and research-based approaches, high-quality instructional materials unlock and support knowledge-building that encourages active learning and leads to dynamic demonstrations of knowledge from students. Furthermore, these materials offer support for educators, equipping them with the necessary tools, content knowledge, pedagogical expertise, and research-based practices to effectively engage students and adapt to diverse community and school contexts. With this comprehensive approach, instructional materials in Maryland have the potential to create transformative learning environments that prepare students from kindergarten through graduation for a future of choice and opportunity.

[1] In 2022, Maryland's student population included 33% Black, 33% White, 22% Latinx and 7% Asian students, as well as 12% English learners, 12% students with disabilities, and an increasing proportion who face economic challenges (Maryland Department of Education).

Document Introduction

This framework serves as a valuable resource for educators and stakeholders across the education sector to identify key criteria in truly high-quality instructional materials. It outlines the essential elements of outstanding curricula and offers clear guidelines on the necessary instructional shifts and educator supports needed to foster meaningful learning experiences for students. To deliver the world-class education that the Blueprint for Maryland's Future envisions, educators and leaders can rely on this framework in service of identifying research-based, high-quality materials that are necessary to provide students with rigorous instruction, nurture spaces that affirm their cultural and linguistic identities, and ensure students' continued progress and success each year.

This framework is grounded in extensive research aimed at defining the content, instructional practice, and instructional design present in high-quality instructional materials. These research-based elements are central to the criteria within this framework and critical to support the kinds of learning experiences that Maryland students deserve.

Despite its strengths as a resource for identifying high-quality instructional materials, there are limitations for how this framework can be used. While the document presents crucial guidelines, it is NOT intended to be exhaustive in addressing all elements of instructional materials and practices needed to create an equitable experience for students. Additionally, this document is NOT a rubric, meaning it does not provide a checklist or a scoring system for evaluation of instructional materials. Instead, it offers guidance on the essential components of high-quality materials, encouraging educators to exercise professional judgment and adapt to their specific educational context. From this framework, a complimentary ELA/literacy rubric has been designed to make these criteria measurable in service of evaluating the quality of instructional materials.

It is also important for educators and leaders to recognize any and all humanizing considerations beyond the framework that may be necessary based on the unique students, contexts of classrooms, and school/district conditions in their review and selection of high-quality materials using this framework. Overall, this framework serves as a roadmap, empowering educators to select and utilize the instructional materials that foster inclusivity, rigor, and relevance, ultimately resulting in enhanced student learning outcomes for all students.

DOCUMENT ORGANIZATION

This document, intended for use when considering K-12 ELA/literacy core instructional materials, is organized into four categories (Designed to Affirm Students; Grade-Level and Standards-Aligned; Instructional Design; and Educator Supports), with domains that highlight Key Criteria within each subsection.

While specific categories have been included for Culturally Responsive-Sustaining Pedagogy and Language Affirming Instruction, related considerations for affirming students are woven throughout the framework. Similarly, considerations for diverse learning needs and Universal Design for Learning have been embedded throughout to reflect the way that these practices must be interlaced in thinking about content, instructional practice and support for educators.

A collection of research and scholarship used to inform this framework is included as an Appendix.

Key Criteria for High-Quality Instructional Materials

DESIGNED TO AFFIRM STUDENTS

Affirming students creates opportunities for cultural and linguistics backgrounds to be an asset and a source of validation in the learning experience. In addition to a foundation of grade-level content, high quality instructional materials must prioritize instructional practices that affirm students' cultural and linguistic backgrounds and support students with a range of diverse learning needs to thrive through mathematics. This includes developing culturally responsive-sustaining learning communities that center who students are, use mathematics as a tool for civic engagement, and connect learning to the world outside the schoolhouse walls. Mathematics instruction must also intentionally affirm students' languages and language practices through a focus on building upon students' multilingualism, ensuring texts support disciplinary language development, and designing language objectives that work in concert with content and mathematics learning. Through these instructional choices, materials have the potential to deepen mathematics learning, cultivate a sense of belonging, and develop students' mathematical identities[2] – to see themselves as a participant in mathematics.

Key Criteria for Culturally Responsive-Sustaining Instruction

- **Affirm and Center Students:** Instructional materials affirm, engage, and center past and current knowledge of Black/African, Indigenous, Brown, and non-Western conceptions of math and highlight multilingualism and non-Western mathematicians and their discoveries. Instructional materials are designed to encourage students to anchor learning in individual experiences, backgrounds, and cultural knowledge to expand their mathematics knowledge and skills.
- **Mathematics as a Tool for Civic Engagement:** Instructional materials consistently include tasks that prompt students to apply and develop their civic engagement skills and examine social contexts and current events, using mathematics to question the world and the current status quo
- **Real World Connections and Relevant Data:** Instructional materials consistently connect with students' lives, future goals, communities, and the world and nurture ways to engage in their own communities and beyond. This includes (all of the following):
    a. use mathematical concepts and tasks to connect to current events;
    b. engage in collaborative tasks and/or projects that involve real-world problem-solving through meaningful interactions with peers and their local communities; and
    c. center and include structures (e.g., tasks, classroom activities, routines, assignments, etc.) to explore mathematical concepts from current events and data relevant to students' lives and communities3 so that students see themselves in the tasks and understand how they relate to their context and promote a sense of belonging.

[2] Aguirre, J., Mayfield-Ingram, K., & Martin, D. B. (2013). The impact of identity in K-8 mathematics learning and teaching: Rethinking equity-based practices. National Council of Teachers of Mathematics, Incorporated, 13-16.

[3] Tate, W. F. (1995). Returning to the root: A culturally relevant approach to mathematics pedagogy. Theory into practice, 34(3), 166-173.

d. include teacher guidance to support students in developing mathematical skills and knowledge relevant to their academic and professional goals.

Key Criteria for Language Affirming Instruction

- **Multilingualism in Mathematics:** Instructional materials are deliberately designed to honor and build upon students' language as an asset, encouraging students to use their linguistic repertoire to communicate with one another via reading, writing, speaking, and listening while engaging in mathematical learning. This includes, within teacher guidance and instructional materials (all of the following):

    a. facilitation and engagement support for students to communicate as they engage in doing the math, making meaning, and collaboratively solving problems;

    b. building mathematical language and content in English and home language(s), including use of social and academic vocabulary, through translanguaging[4] so all students express themselves in a language they are comfortable with while working to learn mathematical content and meet language objectives in the target language; and

    c. making cross-linguistic connections, including identifying and comparing similarities and differences between home language(s) and English (e.g., cognates) or registers and registers of instruction.

- **Language Objectives:** Instructional materials provide explicit alignment between language and content objectives to ensure language goals embedded within standards are being attended to in every lesson. This includes language objectives for both expressive (writing and speaking) and receptive (listening and reading) communication, aligned to the math learning goal.

- **Cognitively Demanding Mathematics:** Instructional materials provide cognitively demanding mathematics tasks that offer multiple research-based entry points and linguistic scaffolds to meet the needs of multilingual learners and students with diverse learning needs.

- **Share Reasoning in Multiple Ways:** Instructional materials include tasks that invite students to share their reasoning in multiple ways and include guidance (e.g., annotations for teachers facilitating the tasks) about encouraging students to transverse between and among different representations (e.g., oral language and pictorial representations, written word and math tools, etc.).

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[4] García, O., Johnson, S. I., & Seltzer, K. (2017). The Translanguaging Classroom: Leveraging Student Bilingualism for Learning. Caslon. ;

For more see https://assets-global.website-files.com/5b43fc97fcf4773f14ee92f3/5cca8e1dbfa8f118e41c578a_Translanguaging%20Strategies%20ELA.pdf

GRADE-LEVEL AND STANDARDS ALIGNED

Grade-level, standards-aligned content serves as a necessary foundation for equitable student experiences in the classroom. Engaging with this rigorous content from kindergarten through graduation sets students on a path to empowered lives, and instructional materials must be designed so that all students have access to this essential work. This includes ensuring all students are empowered by secure engagement with the most important and applicable mathematics of each grade or course; are positioned as mathematical leaders and doers in classrooms; leverage high-quality questions and tasks to practice and tune Standards of Mathematical Practice with content standards, develop their conceptual understanding, procedural skill and fluency, and application; and develop language along with mathematical content knowledge.

Key Criteria for Essential Mathematics

- **Essential Mathematics:** Instructional Materials prioritize the most important and applicable concepts, knowledge, and mathematical skills so that:
    a. there is a Focus on the Major Work of the grade (K-8)[5] and Essential Concepts from Catalyzing Change in HS Mathematics in high school[6] (see appendix); and
    b. students and teachers, using the materials as designed, spend the majority of their time focused on the Essential Mathematics of the grade/course.

Key Criteria for Coherence

- **Consistent Progressions:** Instructional materials are consistent with the progressions in the Standards, by (all of the following):
    a. basing content progressions on the grade-by-grade and course-by-course progressions in the Standards;
    b. giving all students extensive work with grade-level, or high school course-level, problems; and
    c. relating grade-level, or high school course-level, concepts explicitly to prior knowledge from earlier grades or courses.
- **Coherent Connections:** Instructional materials foster coherence through connections within a single grade, or course, where appropriate and where required by the Standards, by (all of the following):
    a. using supporting content to further engage students in the Major Work of the grade in K-8 and supporting content to further engage students in Essential Concepts in high school; and

[5] K–8 Publishers' Criteria for the Common Core State Standards for Mathematics, 9 Apr. 2013, https://www.thecorestandards.org/wp-content/uploads/Math_Publishers_Criteria_K-8_Spring_2013_FINAL1.pdf

[6] Catalyzing Change in High School Mathematics: Initiating Critical Conversations. The National Council of Teachers of Mathematics, Inc., 2018.

b. preserving the focus, coherence, and rigor of the Standards even when targeting specific objectives.

Key Criteria for Rigor and Balance

- **Rigor and Balance:** Instructional materials reflect the aspect(s) of Rigor[7], conceptual understanding, procedural skill and fluency, and/or application, called for by the standards by:
    a. developing students' conceptual understanding of key mathematical concepts, especially where called for in specific content standards or cluster headings;
    b. giving attention throughout the year to procedural skill and fluency; and
    c. allowing teachers and students using the materials as designed to spend sufficient time working with applications that engage students in problem solving.

Key Criteria for Mathematical Practices

- **Practice-Content Connections:** Instructional materials meaningfully integrate Standards for Mathematical Practice[8] with content standards and attend to the full meaning of each practice standard in tasks and problems.
- **Emphasis on Mathematical Reasoning:** Instructional materials support Standards' emphasis on mathematical reasoning through indicating and providing guidance about the opportunities for discourse, communication, problem solving, and modeling.

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INSTRUCTIONAL DESIGN

Instructional materials must attend to research-based instructional practices that support meaningful engagement for all students in order to be deemed high-quality. It is through this intentional design that instructional materials contribute to learning communities where students unlock knowledge; engage with tasks and peers as mathematicians; regularly demonstrate their learning; and experience joy in the math classroom. This type of learning community builds students' mathematical identities, allowing students to see themselves and their peers as mathematical experts, thinkers, and doers in the classroom.

[7] The three aspects of Rigor are not always separate in materials (Conceptual understanding and fluency go hand in hand; fluency can be practiced in the context of applications, and brief applications can build conceptual understanding. Nor are the three aspects of Rigor always together in materials (Fluency requires dedicated practice to that end. Rich applications cannot always be shoehorned into the mathematics topic of the day. And conceptual understanding will not always come along for free unless explicitly taught.)

[8] National Governors Association Center for Best Practices, Council of Chief State School Officers]. (2010). Common core state standards for mathematics. Retrieved from https://learning.ccsso.org/wp-content/uploads/2022/11/Math_Standards1.pdf, 6-8.

Key Criteria for Student Agency

Metacognitive Processes: Instructional materials develop students' metacognitive skills in order to promote understanding of math concepts by directly teaching and supporting students to monitor understanding and progress over time. This includes guidance on:

a. setting goals; self-monitoring growth; and reflecting on the impact of students' choices and ongoing development as mathematical doers, critical thinkers, and communicators;

b. explicit practices to develop students' metalinguistic awareness around how language works in mathematics, language use, and choices connected to mathematical ideas;

c. modeling and developing strategies that support students in making their thinking visible through speaking, writing, or drawing their developing understanding; and

d. supporting students with diverse learning needs in developing metacognitive strategies.

- **Choice:** Instructional materials prompt teachers to provide students with ample time to explore math concepts, during which students are given opportunities to exercise choice in how to spend time, who to spend it with, and what materials are used.
- **Multiple Entry Points to Complex Tasks:** Instructional materials include tasks that are complex, with multiple entry points (e.g., allows for multiple solution strategies, encourages use of multiple representations) that promote collaboration and different ways of thinking and explaining.
- **Authentic Engagement as a Mathematician:** Instructional materials promote productive struggle and the mathematical modeling process through quality math tasks that are sequenced to build conceptual understanding and procedural skill and fluency, prioritize inquiry, provide opportunities to take risks, allow for rough draft thinking and multiple approaches, invite the use of math tools, and utilize mistakes for learning so that students engage in collaborative learning.
- **Collaborative Learning:** Instructional materials engage all students in collaborative learning through a variety of research-based routines, structures, and tasks that allow for whole group, small group, and independent thinking. Materials explicitly plan for students to demonstrate their curiosity and share their tentative thinking; ask questions; and adjust their understandings by listening to and building on one another's shared ideas.

Key Criteria for Progress Monitoring and Supporting Students

- **Supports & Scaffolds:** Instructional materials are designed to support a variety of student strengths and diverse learning needs in ways that are supported by research and maintain attention to grade-level content alongside practice standards. This includes (all of the following):

a. scaffolds and supports that are designed based on mathematical learning progressions and the coherence of math concepts across and within grades and courses;

b. guidance on identifying scaffolds and appropriate supports that build on students' mathematical thinking, ideas, and experiences; and

c. content-specific guidance on identifying and addressing potential individual students needs so that supports, scaffolds, and extensions can be effectively differentiated, including adjustments to content, process, or product.

- **Simultaneous Mathematical Meaning Making and Language Development:** Instructional materials include intentional language learning opportunities alongside appropriate, research-based supports for multilingual learners and students with diverse learning needs in order to develop mathematical meaning-making and language simultaneously. This includes questions for students to raise metalinguistic awareness of how language works in math and integrating language standards alongside mathematical content standards[9].
- **Relevant Contexts:** Instructional materials provide contextualized tasks and problems, and opportunity to contextualize tasks and problems, that incorporate students' everyday lives, families, and communities' ways of knowing, including their language and culture[10].
- **Mathematical Discourse:** Instructional materials are designed to allow for students to shape the mathematical discourse, through specifying opportunities for students to listen to, share with, and build on peer mathematical thinking.
- **Practice opportunities and resources:** Instructional materials include well-designed, grade-level practice opportunities that focus on essential mathematics and align within the progression. These opportunities are designed to include (all of the following) considerations:
    a. a variety of modes and meaningful contexts (e.g., games, puzzles, whiteboards, card sorts, interactive problem solving);
    b. low floor, high ceiling: practice should represent a flexible range of access and challenge that allow students to engage and practice across a spectrum of problems;
    c. purpose over quantity: practice should have an intentional and clear connection to the current learning progression and involve students in reflection and self-assessment, through the provision of solutions (calculations, representations, and/or writing) with reflection prompts, to mark progress towards goals; and
    d. fluency: practice materials should be designed in support of the deep connections between conceptual understanding and fluency.
- **Progress Monitoring:** Instructional materials embed frequent opportunities for students to demonstrate understanding of grade-level mathematical concepts using their existing language resources[11]. Instructional materials also embed resources and frequent opportunities to monitor and respond to students' understanding of grade-level mathematics. Materials

[9] Moschkovich, J. (2012). Mathematics, the Common Core, and Language: Recommendations for Mathematics Instruction for ELs Aligned with the Common Core. Retrieved from https://ul.stanford.edu/sites/default/files/resource/2021-12/02-JMoschkovich%20Math%20FINAL_bound%20with%20appendix.pdf

[10] Celedón-Pattichis, S., Borden, L. L., Pape, S. J., Clements, D. H., Peters, S. A., Males, J. R., ... & Leonard, J. (2018). Asset-based approaches to equitable mathematics education research and practice. *Journal for Research in Mathematics Education*, *49*(4), 373-389.

[11] "ELSF: Math Guidelines 5.15." ELSF | Math Guidelines, www.elsuccessforum.org/math-guidelines. Accessed 11 July 2023.

demonstrate how to diagnose critical student needs and draw clear connections to integrating supports and prioritizing instruction. This includes (all of the following):

a. embedded and consistent formative assessment practices for mathematical content, mathematical literacy, and language learning;

b. varied ways and multiple means of using formative data (including opportunities, beyond calculation alone, to explain, write, represent, self-reflect, and connect ideas) to demonstrate students' mathematical thinking and to make instructional decisions based on students' mathematical thinking; and

c. regular monitoring of grade-level mathematics development.

- **Meaningful Feedback:** Instructional materials and teacher guidance provide frequent opportunities and facilitation notes on how to provide meaningful feedback to advance mathematical understanding and language. This includes (all of the following):

a. peer and teacher cycles of feedback, including communicating progress with affirming evidence of mathematical progress;

b. normalizing mistake making and affirming effort and growth;

c. providing guidance for explicit, timely, informative, and accessible formative feedback to address partial solutions and alternative thinking in ways that allow learners to monitor their own progress effectively and to use that information to guide their own effort and practice without sacrificing their math confidence;

d. focusing students' attention on sense-making and/or metacognitive processes; and

e. guidance on how and when to collect data, as well as how to respond to specific student strengths and needs.

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EDUCATOR SUPPORTS

To promote facilitation of meaningful learning experiences for all students, instructional materials ensure effective support for educators in their instructional practices and use of supplemental resources. Throughout the instructional materials, explicit tools and resources focus on enhancing educators' depth of mathematical knowledge for teaching, on utilizing pedagogical content knowledge in planning for instruction, and on practicing responsive teaching to build on or extend students' mathematical thinking. These tools and resources also encourage reflective practices among educators, including the examination of their own identities as well as identifying places where teacher actions may contribute to building positive mathematical identities in their students. In addition, resources are thoughtfully designed for ease of use and fit to community context.

Key Criteria for Educator Knowledge

- **Examine Self**: Instructional materials support teachers in examining their own identities, biases, and belief systems to help them understand how these factors might influence instructional choices and the lens through which they interpret student thinking. This may include reflection prompts, examples of educator thinking, or embedded professional learning.
- **Students' Linguistic and Cultural Assets:** Instructional materials support educators to see and to leverage students' linguistic and cultural assets, approaching these assets with a disposition of curiosity and appreciation. This includes prompting educators to learn and integrate the knowledge, strengths, and resources that students, families, and the community so that mathematics instruction leverages students' linguistic and cultural assets - especially those who have been historically marginalized[12]. This includes connecting to and bringing in math topics and ideas from the backgrounds of students, drawing from students' home and everyday language to learn mathematics[13], and building and strengthening relationships that elicit and center these assets to bridge and propel relevance of learning.
- **Supporting Language Development for all Learners:** Instructional materials build educators' understanding of research-based practices to support language development for all learners, especially for multilingual learners and students with diverse learning needs, including (all of the following):
    a. developing explicit language objectives for communication about mathematics[14];
    b. building knowledge of students' language development and language development standards, as connected to the mathematics of the lesson or unit;
    c. enacting math language routines[15] to foster mathematical discourse and communication amongst students;
    d. providing examples of sample student responses, in the context of actual mathematics tasks, with a range of language proficiency[16];
    e. suggestions of ways to capture student progress from everyday language to language for more formal academic and mathematical purposes; and
    f. guidance on what to look for, listen for, questions to ask, and/or feedback to give when supporting multilingual learners.

[12] Aguirre, J., Mayfield-Ingram, K., & Martin, D. B. (2013). The impact of identity in K-8 mathematics learning and teaching: Rethinking equity-based practices. National Council of Teachers of Mathematics, Incorporated.

[13] Moschkovich, J. (2013). Principles and guidelines for equitable mathematics teaching practices and materials for English language learners. Journal of Urban Mathematics Education, 6(1), 45-57

[14] Gottlieb, M., & Ernst-Slavit, G. (2014). Academic language in diverse classrooms: Definitions and contexts. Corwin.

[15] Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). Principles for the design of mathematics curricula: Promoting language and content development. https://ell.stanford.edu/sites/default/files/u6232/ULSCALE_ToA_Principles_MLRs__Final_v2.0_030217.pdf

[16] "ELSF: Math Guidelines 5.13, 5.14, 2.5." *ELSF | Math Guidelines*, www.elsuccessforum.org/math-guidelines. Accessed 11 July 2023.

- **Supporting Mathematical Development:** Instructional materials deepen educators' mathematical knowledge for teaching through building educators' understanding of research-based practices to support routines for reasoning, inquiry-based approaches, and structures that develop and affirm positive math mindsets during the process of supporting all students in understanding grade-level mathematics. This includes (all of the following):
    a. teacher guidance on multiple math strategies and the ways in which those approaches represent different, but equally valid, conceptions of the same mathematical idea(s); and
    b. guidance on what to look for, listen for, questions to ask, and/or feedback to give so that mathematical inquiry and reasoning is student-led.
- **Mathematical Discourse:** Instructional materials are designed to foster educator facilitation of mathematical discourse shaped by students, through specifying opportunities for students to listen to, share with, and build on peer mathematical thinking. This includes guidance on structuring activities to support students in:
    a. sharing their own mathematical thinking with their peers;
    b. engaging with their peers' mathematical thinking;
    c. reflecting on and articulating their own understanding of their peers' mathematical perspectives;
    d. building on and extending their peers' mathematical ideas; and
    e. providing feedback to their peers on their mathematical reasoning.
- **Collectivist Approach:** Instructional materials provide teacher guidance that counters traditional math structures of individualism and competition by structuring the doing of mathematics through collaboration.

Key Criteria for Usability

- **Design and Functionality:** Instructional materials are designed to support ease of student and teacher use. This includes (all of the following):
    a. materials are scalable and accessible, and curriculum can be disseminated in a way that ensures equitable student, teacher, and community access;
    b. a visually appealing design with an organized and logical format;
    c. materials that are appropriately paced;
    d. clear and concise educator-facing guidance that enables educators to prepare lessons in a timely manner; and
    e. a variety of ways to engage with the content, including leveraging current technology and tools (e.g., online graphing calculators, digital manipulatives, etc.).
- **Adaptability for Context:** Instructional materials contain materials and/or meaningful suggestions for how to adapt for district, school, and/or classroom context. This may include varied selections for topics under study; opportunities to modify tasks to connect to local

resources, organizations, or issues; or varied pacing suggestions based on number of school days or minutes of instruction.

- **Program Coherence**: Core instructional materials guide the use of additional supplemental mathematics materials (e.g., interventional materials), in content and approach. Use of supplemental materials support students in accessing the grade-level mathematics content that is concurrently happening in core instruction.

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Table 1. Progress to Algebra in Grades K–8

K	1	2	3	4	5	6	7	8
Know number names and the count sequence Count to tell the number of objects Compare numbers Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from Work with numbers 11-19 to gain foundations for place value	Represent and solve problems involving addition and subtraction Understand and apply properties of operations and the relationship between addition and subtraction Add and subtract within 20 Work with addition and subtraction equations Extend the counting sequence Understand place value Use place value understanding and properties of operations to add and subtract Measure lengths indirectly and by iterating length units	Represent and solve problems involving addition and subtraction Add and subtract within 20 Understand place value Use place value understanding and properties of operations to add and subtract Measure and estimate lengths in standard units Relate addition and subtraction to length	Represent & solve problems involving multiplication and division Understand properties of multiplication and the relationship between multiplication and division Multiply & divide within 100 Solve problems involving the four operations, and identify & explain patterns in arithmetic Develop understanding of fractions as numbers Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects Geometric measurement: understand concepts of area and relate area to multiplication and to addition	Use the four operations with whole numbers to solve problems Generalize place value understanding for multi-digit whole numbers Use place value understanding and properties of operations to perform multi-digit arithmetic Extend understanding of fraction equivalence and ordering Build fractions from unit fractions by applying and extending previous understandings of operations Understand decimal notation for fractions, and compare decimal fractions	Understand the place value system Perform operations with multi-digit whole numbers and decimals to hundredths Use equivalent fractions as a strategy to add and subtract fractions Apply and extend previous understandings of multiplication and division to multiply and divide fractions Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition Graph points in the coordinate plane to solve real-world and mathematical problems*	Apply and extend previous understandings of multiplication and division to divide fractions by fractions Apply and extend previous understandings of numbers to the system of rational numbers Understand ratio concepts and use ratio reasoning to solve problems Apply and extend previous understandings of arithmetic to algebraic expressions Reason about and solve one-variable equations and inequalities Represent and analyze quantitative relationships between dependent and independent variables	Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers Analyze proportional relationship and use them to solve real-world and mathematical problems Use properties of operations to generate equivalent expressions Solve real-life and mathematical problems using numerical and algebraic expressions and equations	Work with radical and integer exponents Understand the connections between proportional relationships, lines, and linear equations** Analyze and solve linear equations and pairs of simultaneous linear equations Define, evaluate, and compare functions Use functions to model relationships between quantities

*Indicates a cluster that is well thought of as part of a student's progress to algebra, but that is currently not designated as Major by one or both of the assessment consortia in their draft materials. Apart from the asterisked exception, the clusters listed here are a subset of those designated as Major in both of the assessment consortia's draft documents. ** Depends on similarity ideas from geometry to show that slope can be defined and then used to show that a linear equation has a graph which is a straight line and conversely.

K–8 Publishers' Criteria for the Common Core State Standards for Mathematics, 9 Apr. 2013, achievethecore.org/content/upload/Math_Publishers_Criteria_K-8_Spring_2013_FINAL.pdf.

Research & Scholarship Supporting the Framework

A robust research and scholarship base underpins this framework. For more information about research-supported practice, see Student Achievement Partners' Essential X Equitable Instructional Practice Framework™.

DESIGNED TO AFFIRM STUDENTS

Culturally Responsive-Sustaining

Gutstein, E., & Peterson, B. (Eds.). (2005). Rethinking mathematics: Teaching social justice by the numbers. Rethinking Schools.

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GRADE-LEVEL AND STANDARD ALIGNED

Focus on Essential Mathematics

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Rigor and Balance

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Mathematical Practices

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National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). Common Core State Standards for Mathematics. Washington, DC: Authors.

Smith, M. S., & Stein, M. K. (2022). *5 Practices for orchestrating productive mathematics discussions* (2nd ed.). National Council of Teachers of Mathematics, Inc.

INSTRUCTIONAL DESIGN

Student Agency

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